Final exam of Particle Physics
Monday February 5th 2024

Duration: 3 hours
5 printed pages

Allowed material: PDG booklet, simple calculator.
Solve on two separate sheets exercises I-III and exercises IV-V.

Approximate duration per exercise:

Ex. I:	10 min.	Ex. II:	60 min.
Ex. III:	20 min.	Ex. IV:	25 min.
Ex. V:	65 min.		

Exercise I
Short questions on the lectures and general understanding

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. Explain what is a fragmentation function, and why it has a perturbative and a non-perturbative components. How are these two components obtained in practice?
2. Comment on the branching fractions of the decays $D^+ \to \ell\nu_\ell$, where ℓ represents e, μ or τ, and explain their hierarchy.

Exercise II
H and Z bosons production at the LHC

We consider the production at the LHC of the Higgs boson (H) and the Z boson. We use the symbol R to refer generically to either of the two. We thus consider:

$$AB \to R$$

where A and B are the two incoming partons from p-p collisions. We neglect the masses of all fermions and all partons. We define:

- 4-momenta of proton containing A, B: $q_A \equiv (E, 0, 0, +E)$, $q_B \equiv (E, 0, 0, -E)$,
- 4-momenta of parton A, B: $\hat{p}_A \equiv (E_A, 0, 0, +E_A)$, $\hat{p}_B \equiv (E_B, 0, 0, -E_B)$,
- $\hat{s} \equiv (\hat{p}_A + \hat{p}_B)^2$, $s \equiv (q_A + q_B)^2$.

Parts 1-3 are independent. For simplicity, we will consider for partons A, B:

- $A \in [u, d, s, g]$,
- $B \in [\bar{u}, \bar{d}, \bar{s}, g]$.

1. *Phase space in parton density function*

   (a) Express $\hat{p}_A$ and $\hat{p}_B$ as a function of q_A and q_B and their corresponding Bjorken variables x_A and x_B. What is the physical meaning of x_A and x_B?

   (b) Neglecting the proton and quark masses, give the expression of $\hat{s}$ as a function of s.

   (c) Give the expressions of E_R and pz_R as functions of x_A, x_B and E (the proton-beam energy).

   (d) Obtain the expression of the rapidity Y_R of the boson R as a function of x_A, x_B and E. As a reminder Y_R is given by

   $$Y_R = \frac{1}{2}\ln\left(\frac{E_R + p_{zR}}{E_R - p_{zR}}\right)$$

   (e) What is the typical value of $\hat{s}$ required to produce a boson R?

   (f) Using the expression of $\hat{s}$ vs s (question A-2) and Y_R (question A-4), demonstrate that:

   $$x_A = \frac{M_R}{\sqrt{s}}e^{Y_R} \quad \text{and} \quad x_B = \frac{M_R}{\sqrt{s}}e^{-Y_R}\,. \tag{1}$$

2. *H boson production at the LHC (all questions are independent unless stated otherwise).*

   The dominant Higgs boson production is coming from the process $gg \to H$.

   (a) Draw the corresponding Feynman diagram. In the case of a loop diagram, give the names of the particles circulating in the dominant loop (explain your choice).

   (b) Using $m_H = 125$ GeV, from Eq. 1, what are the values of x_A, x_B, assuming $Y_H \approx 0$ for $\sqrt{s} = 8$ TeV and $\sqrt{s} = 13$ TeV?

   (c) From Fig. 1 and the previous question, predict numerically (and explain your reasoning) the ratio

   $$R_H = \frac{\sigma(gg \to H; \sqrt{s} = 13 \text{ TeV})}{\sigma(gg \to H; \sqrt{s} = 8 \text{ TeV})}$$

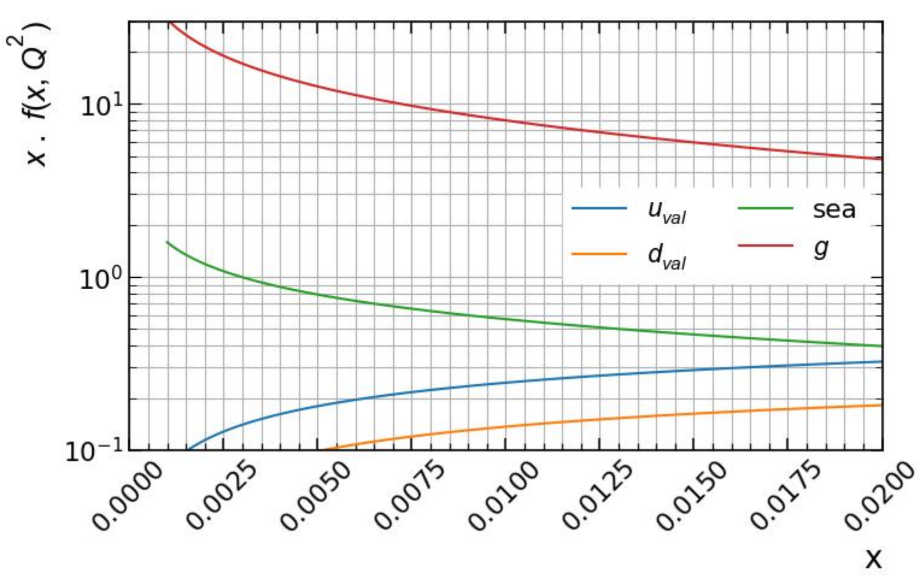

Figure 1: Parton density function for $Q^2 \approx (100 \text{ GeV})^2$

(d) At 13 TeV, the cross section $\sigma(gg \to H; \sqrt{s} = 13\text{ TeV}) \approx 50$ pb. The total integrated luminosity collected by an experiment at the LHC during Run2 is $L_{\text{Run2}} = 137\text{ fb}^{-1}$. What is the total number of Higgs bosons produced at this LHC experiment during Run2? The branching fraction $\mathcal{B}(H \to \gamma\gamma) \approx 0.2$ %. How many $H \to \gamma\gamma$ were produced during Run2?

(e) Assuming $m_b/m_c \approx 3.4$ and $\mathcal{B}(H \to b\bar{b}) \approx 60$ %, predict the branching fraction $\mathcal{B}(H \to c\bar{c})$ at leading order.

3. *Z boson production at the LHC*

We remind you that the coupling Zff is given in the SM by:

$$c_{Zff} = \frac{g}{\cos\theta_W}\left(I_3^f - Q_f \sin^2\theta_W\right). \tag{2}$$

For numerical computations, use the value $\sin^2\theta_W = 0.23$.

(a) Define the quantities g, θ_W in the SM.

(b) What are the values of I_3^f and Q_f for the chiral fields u_L, u_R, s_L, s_R?

(c) What are the potential Z boson production mechanisms at LO in p-p collisions? Draw the Feynman diagram for one of them (specify the nature of the A and B partons).

(d) Assuming $x_A = x_B = 0.0075$ and the convention on the A and B in the introduction (*i.e.* A is a quark or gluon, while B is an antiquark or gluon), is the A parton more likely to be a valence or a sea parton? Same question for parton B.

(e) Assuming $x_A = x_B = 0.0075$ what is the dominant production mechanism taking into account only PDFs (use Fig. 1 and give a qualitative approximate ratio of the different production mechanisms)?

(f) What is the dominant production mechanism from the point of view of couplings?

Exercise III

Z boson couplings

The coupling of the Z boson to fermions is given in Eq. 2. We consider the production of the Z boson at the SLC collider and its subsequent decay to a pair of muons. The SLC collider had a polarized electron beam, and in this exercise, we consider only L-helicity electrons. The reaction is therefore:

$$e_L^- e^+ \to Z \to \mu^+\mu^-$$

We consider the reaction in the e^-e^+ centre of mass. We define the $\vec{z}$ axis as the beam axis oriented in the direction of the e_L^- beam. We denote as θ the angle between the $\vec{z}$ axis and the direction of the μ^- particle.

1. What is the helicity of the e^+ particle (justify)?

2. Using the Wigner-matrix formalism, extract the angular component of the amplitudes of production for the $\mu_L^-\mu_R^+$, $\mu_R^-\mu_R^+$, $\mu_R^-\mu_L^+$, $\mu_L^-\mu_L^+$ final states.

3. Determine the angular distribution of the total cross section $\sigma(e_L^-e^+ \to Z \to \mu^+\mu^-)$ as a function of $\cos\theta$?

4. Propose an experimental method to measure $\sin^2\theta_W$ at SLC.

Change sheet here

Exercise IV

Allowed and forbidden processes, Feynman diagrams

For each of the processes below, determine whether it is allowed or forbidden in the standard model. For the forbidden processes, explain why they are forbidden, giving *all the possible reasons* (here you are not required to take into account multiplicative quantum numbers and angular momentum). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Give all the relevant arguments you find to justify the choice of interaction, and, when applicable, name the topology of the Feynman diagram. Note on the diagram the names of all real and virtual particles. When relevant, indicate near the vertex the CKM matrix elements that contribute and give their orders of magnitude in terms of $\lambda = \sin\theta_c$ (θ_c is the Cabibbo angle). Then, indicate the overall magnitude of the diagram as a power of λ. In the case of Penguin or box diagrams, do this for the dominant intermediate quarks. In general, tree processes are favoured compared to penguin or box processes with comparable CKM factors. Thus, try to privilege tree diagrams when several topologies are possible. Also, try to privilege colour allowed to colour suppressed diagrams, when applied.

1. $B^0 \to \pi^+\pi^-$
2. $K^{*0}(892) \to K^+\pi^-$
3. $J/\psi \to n\tau^+\tau^-\nu_\mu$
4. $\Lambda_b^0 \to \Lambda K^0 \overline{K}^0$
5. $B_s^0 \to \mu^+\mu^-\mu^+\mu^-$

Exercise V

Radiative B^0 meson decays

In this exercise, we will study several aspects of the radiative decays

$$B^0 \to K^{*0}(892)\gamma\,,$$
$$B^0 \to \rho^0(770)\gamma$$

and their *CP*-conjugate processes. In the following, $K^{*0}(892)$ and $\rho^0(770)$ are denoted as K^{*0} and ρ^0, respectively.

Parts 1-5 below are mostly independent.

1. Kinematics and phase-space factors
   (a) For the reactions given above, let p_K (p_ρ) be the momentum of the K^{*0} (ρ^0) in the centre-of-mass frame. Express p_K and p_ρ as functions of the masses of the particles involved. Compute the numerical values of these momenta.
   (b) Express the ratio of the phase-space factors in the two reactions as a function of p_K and p_ρ. Compute the numerical value and comment.
2. Feynman diagrams and branching fractions
   (a) Draw the dominant radiative-penguin diagrams of the two decays, with a dominant virtual top (or anti-top) quark. When relevant, indicate near the vertex the CKM matrix element that contributes. Here, we will take the quark-content of the ρ^0 as $d\bar{d}$.

(b) Make a *qualitative* comment on the contribution of other diagrams in these two decay processes. Explain why the $u\bar{u}$ component of the ρ^0 is not considered in the corresponding diagram.

(c) Express the ratio of branching fractions $\mathcal{B}(B^0 \to K^{*0}\gamma)/\mathcal{B}(B^0 \to \rho^0\gamma)$, as a function of the CKM matrix elements, neglecting any difference in phase-space factors between the two decays. Then, express the ratio as a function of the parameters A, λ, ρ and η of the Wolfenstein parameterisation at $\mathcal{O}(\lambda^3)$. Compute the numerical value of this ratio and compare it to that of the actual measured branching fractions from the PDG. Comment.

3. Decay products of the ρ^0 and K^{*0} and event reconstruction

   (a) Of all the decay modes of the ρ^0 and K^{*0} mesons, which ones would be the simplest to reconstruct? Explain your arguments.

   (b) With the K^{*0} decay mode that you chose in the previous question, explain in a few sentences how would you reconstruct events in the $B^0 \to K^{*0}\gamma$ mode, commenting on relevant invariant-mass spectra.

   (c) Estimate the branching fractions of K^{*0} decays into $K^+\pi^-$ and $K^0\pi^0$, where the K^0 is observed as a K^0_S. Explain your reasoning clearly.

4. Study of the quantum number *CP*

   Along with brief explanations, obtain the *CP* eigenvalues of

   (a) The ρ^0, independently of any particular decay process;

   (b) the $K^0_S\pi^0$ system in the decay $K^{*0} \to K^0_S\pi^0$, using the approximation that K^0_S is a *CP* eigenstate;

   (c) the system $\rho^0\gamma$ in the decay$B^0 \to \rho^0\gamma$;

   (d) the system $K^0_S\pi^0\gamma$ in the cascade of decays $B^0 \to K^{*0}(\to K^0_S\pi^0)\gamma$.

5. Photon polarization and *CP* violation.

   (a) Considering only the hard process $b \to s\gamma$ at the quark level, and neglecting the s-quark mass, show that, in the SM, the photon in this process is left-handed (and right-handed for the conjugate process). This will be the same for $b \to d\gamma$ transitions.

   In the following we consider that, in the two processes under scrutiny, the photon is predominantly left-handed in $\overline{B}^0$ decays and right-handed in B^0 decays.

   (b) Explain why it is so only *predominantly* and not exclusively.

   (c) Without taking into account the handedness of the photon in these processes (for instance, if the photon was a scalar particle), which types of *CP* violation would it be possible to study in $B^0 \to \rho^0\gamma$ decays? Explain.

   (d) In the case of the cascade decay $B^0 \to K^{*0}(\to f)\gamma$, where f is a decay mode of the K^{*0}, give an example of f for which the answer to the previous question is the same. Briefly explain your arguments.

   (e) How does this change when considering the photon handedness, and what behaviour would you expect in this case for the *CP* asymmetry in $B^0 \to \rho^0\gamma$ decays?