Final exam of Particle Physics
Monday February 5th 2024

Duration: 3 hours
5 printed pages

Allowed material: PDG booklet, simple calculator.
Solve on two separate sheets exercises I-III and exercises IV-V.

Exercise I
Short questions on the lectures and general understanding

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. Explain what is a fragmentation function, and why it has a perturbative and a non-perturbative components. How are these two components obtained in practice?
   Elements of the answer

   - Fragmentation function: probability density function of a variable relating quark - hadron kinematics.
   - Examples of such variables: the ratio of the energy/momentum of a hadron containing a heavy quark and that of the initial heavy quark after being produced in a collision, or the ratio of energies of a hadron (quark) after (before) the hadronisation routine in a Monte-Carlo generator is called.
   - When a quark is created it radiates gluons and gradually loses energy. When the quarks energy becomes comparable to Λ_{QCD} the quark hadronises with other quarks created in its vicinity. As long as the energy scale is $\ll \Lambda_{\mathrm{QCD}}$, α_s is small and the gluon radiation process, and thus the corresponding part of the fragmentation function, can be described using a perturbative approach (QCD computation or Monte-Carlo parton shower). When the energy gets closer to Λ_{QCD}, α_s becomes large and the perturbative approach breaks. Thus this part of the fragmentation process has to be described by non-perturbative models.
   - The two parts are supposed to factorize and are convoluted to get the observed fragmentation function.

2. Comment on the branching fractions of the decays $D^+ \to \ell\nu_\ell$, where ℓ represents e, μ or τ, and explain their hierarchy.

   Elements of the answer

   - This case follows exactly the same logic as the comparison of the π^+-meson decay rates into $e^+\nu_e$ and $\mu^+\nu_\mu$
   - The weak interaction, via which proceeds the decays under scrutiny (as seen for example by the presence of neutrinos), couples only left-handed chirality fermions and right-handed chirality antifermions.
   - In the ultra-relativistic limit, a particle's helicity matches its chirality. As the particle becomes less relativistic, a particular helicity contains a greater proportion of the opposite chirality.
   - Similar to the π^+ decay, here, the D^+ meson, with $J = 0$, decays into two $J = 1/2$ leptons that must possess the same helicities to conserve the angular momentum.
   - Since only left-handed neutrinos exist, the charged lepton must be left-handed as well. However, this becomes less probable as the positively charged lepton becomes more relativistic.
   - This explains the observed hierarchy: $\mathrm{BF}(D^+ \to \tau^+\nu_\tau) > \mathrm{BF}(D^+ \to \mu^+\nu_\mu) > \mathrm{BF}(D^+ \to e^+\nu_e)$, despite the opposite hierarchy of the phase-space factor. The τ in this decay is relatively non-relativistic, while the electron is highly ultra-relativistic.

Exercise II

H and Z bosons production at the LHC

1. *Phase space in parton density function* This part was treated as exercise during the course

   (a) $\hat{p}_i = x_i \times q_i$. x_i refers to the Bjorken variable of parton i which physically corresponds to the fraction of the nucleon-i 3-momentum carried by parton i.

   (b) $\hat{s} = (\hat{p}_A + \hat{p}_B)^2 = \hat{p}_A^2 + \hat{p}_B^2 + 2\hat{p}_A\hat{p}_B = 2x_Ax_Bp_Ap_B$, since we neglect the masses of parton A and B, *i.e.* $\hat{p}_A^2 \approx 0$ and $\hat{p}_B^2 \approx 0$. Similarly we can show that $s = (q_A + q_B)^2 \approx 2q_Aq_B$. Thus we get that
   $$\hat{s} = x_Ax_B\ s$$

   (c) $E_R = E_A + E_B = (x_A + x_B)\ E$. Similarly $p_{ZR} = p_{ZA} + p_{ZB} = (x_A - x_B)\ E$.

   (d) Therefore:
   $$\begin{aligned} Y_R &= \frac{1}{2}\ln\left(\frac{E_R + p_{zR}}{E_R - p_{zR}}\right) \\ &= \frac{1}{2}\ln\left(\frac{(x_A + x_B)E + (x_A - x_B)E}{(x_A + x_B)E - (x_A - x_B)E}\right) \\ &= \frac{1}{2}\ln\frac{x_A}{x_B} \end{aligned} \tag{1}$$

   (e) To produce a boson R, we typically need $\hat{s} \approx M_R^2$.

   (f) By multiplying $\frac{x_A}{x_B} = e^{2Y_R}$ with $x_Ax_B = \hat{s}/s \approx M_R^2/s$, we obtain the desired results:
   $$x_A = \frac{M_R}{\sqrt{s}}e^{Y_R} \quad \text{and} \quad x_B = \frac{M_R}{\sqrt{s}}e^{-Y_R}. \tag{2}$$

2. *H boson production at the LHC (all questions are independent unless stated otherwise).*

   The dominant Higgs boson production is coming from the process $gg \to H$.

   (a) See course. The dominant process is a triangle diagram with a top circulating in the loop. The top loop dominates over all the others because the coupling of the Higgs boson to fermion f is proportional to m_f (m_f^2 for the cross section) and the loop is extremely more massive than any other fermions.

   (b) Because $Y_H \approx 0$, we get $x \equiv x_A = x_B$:
   $$\begin{aligned} x_{8\text{TeV}} &= 125/8000 \approx 0.016 \\ x_{13\text{TeV}} &= 125/13000 \approx 0.010 \end{aligned} \tag{3}$$

   (c) The cross section is given by $\iint dx_Adx_Bf_A(x_A)f_B(x_B)\hat{\sigma}(\hat{s} = x_Ax_Bs)$. Since we consider distribution of x_A and x_B to be close to a Dirac distribution, we get that:
   $$\begin{aligned} R_H &\equiv \frac{\sigma(gg \to H; \sqrt{s} = 13\ \text{TeV})}{\sigma(gg \to H; \sqrt{s} = 8\ \text{TeV})} \\ R_H &= \frac{f_A(x_{13\text{TeV}})f_B(x_{13\text{TeV}})}{f_A(x_{8\text{TeV}})f_B(x_{8\text{TeV}})} \\ &= \frac{[f_\text{g}(x_{13\text{TeV}})]^2}{[f_\text{g}(x_{8\text{TeV}})]^2} \\ &= \frac{(8/0.010)^2}{(6/0.016)^2} \\ &\approx 4.6 \end{aligned} \tag{4}$$

(d) Number of Higgs bosons produced at the LHC over the course of Run2: $N_H = \sigma(gg \to H; \sqrt{s} = 13\text{ TeV}) \times L_{\text{Run2}} \approx (50\text{ pb}) \times (137000\text{ pb}^{-1}) \approx 6.9\ 10^6$ Number of Higgs bosons decaying in the diphoton channel: $N_{H\gamma\gamma} = N_H \times (B)(H \to \gamma\gamma) \approx 13700$.

(e) The branching fraction into $c\bar{c}$ can be obtained from the one into $b\bar{b}$ via:

$$\begin{aligned}
\mathcal{B}(H \to c\bar{c}) &= \frac{\Gamma_{c\bar{c}}}{\Gamma_{\text{tot}}} \\
&= \frac{\Gamma_{c\bar{c}}}{\Gamma_{b\bar{b}}} \times \frac{\Gamma_{b\bar{b}}}{\Gamma_{\text{tot}}} \\
&= \frac{\Gamma_{c\bar{c}}}{\Gamma_{b\bar{b}}} \times \mathcal{B}(H \to b\bar{b}) \\
&\approx \left(\frac{m_c}{m_b}\right)^2 \times \mathcal{B}(H \to b\bar{b}) \approx 5.2\ \%
\end{aligned} \tag{5}$$

The last line is obtained by assuming that the b and c quarks can be considered as massless with respect to the Higgs boson mass. Therefore in the partial decay width, the phase space term can be neglected and solely the coupling plays a role. Since the coupling of the Higgs boson to fermions (named Yukawa coupling) is proportional to the fermion mass, the partial widths are proportional to the fermion mass squared!

3. *Z boson production at the LHC*

We remind you that the coupling Zff is given in the SM by:

$$c_{Zff} = \frac{g}{\cos\theta_W}\left(I_3^f - Q_f \sin^2\theta_W\right). \tag{6}$$

For numerical computations, use the value $\sin^2\theta_W = 0.23$.

(a) g is the coupling constant of $SU(2)_L$ and θ_W the Wigner angle which corresponds of the mixing between the neutral bosons of $SU(2)_L \times U(1)_Y$ which give after the EWK symmetry breaking the Z and γ bosons.

(b) The values of I_3^f and Q_f are iven in Tab. 1.

Field f	u_L	u_R	s_L	s_R
I_3^f	+1/2	0	-1/2	0
Q_f	+2/3	+2/3	-1/3	-1/3

Table 1: Values of the isospin I_3^f and charge Q_f of the different fields

(c) Since we consider that the proton is composed solely of u, d and s quarks, the different production of the Z boson at LO are coming from $u\bar{u} \to Z$, $d\bar{d} \to Z$, $s\bar{s}t\o Z$. The diagram is just a Zff vertex.

(d) Because we are at low x sea quarks dominates over valence quarks in the nucleon (see the relevant figure) for the A parton. We get the relative contributions of each pdf from the figure: $sea : u_{\text{val}} : d_{\text{val}} \approx 6.5 : 2.1 : 1.1$ (even more approximated values such as $7 : 2 : 1$ are accepted as answer). The b parton is always a sea quark since protons do no contain valence anti-quarks.

(e) Therefore the productions can be ranked as (from the most important one to the least important) $u\bar{u} \to Z : d\bar{d} \to Z : s\bar{s} \to Z^0$ with the respective production ratios $6.5 + 2.1 : 6.5 + 1.1 : 6.5 \approx 1.3 : 1.1 : 1$. Because the sea quark production dominates, the 3 modes are of similar importance.

(f) From the coupling point of view, one has to remimeber that there are in fact two different productions for each single process ($\sigma(q\bar{q} \to Z) = \sigma(q_L\bar{q}_R \to Z) + \sigma(q_R\bar{q}_L \to Z) \propto c^2_{Zf_L\bar{f}_R} + c^2_{Zf_R\bar{f}_L}$ which arise from 2 different couplings. Thus, the 3 different productions are proportional to:

$$\begin{aligned}\sigma(u\bar{u} \to Z) &\propto (+1/2 - 2/3\sin^2\theta_w)^2 + (0 - 2/3\sin^2\theta_w)^2 = 0.144 \\ \sigma(d\bar{d} \to Z) &\propto (-1/2 + 1/3\sin^2\theta_w)^2 + (0 + 1/3\sin^2\theta_w)^2 = 0.185 \\ \sigma(s\bar{s} \to Z) &\propto (-1/2 + 1/3\sin^2\theta_w)^2 + (0 + 1/3\sin^2\theta_w)^2 = 0.185\end{aligned} \tag{7}$$

giving the relative proportions $u\bar{u} \to Z : d\bar{d} \to Z : s\bar{s} \to Z = 0.8 : 1 : 1$, roughly compensating the pdf effect so the $u\bar{u}$ and $d\bar{d}$ productions are of similar order.

Exercise III

Z boson couplings

1. Because vector and pseudo-vector interactions couple left chiral fields together (or right chiral fields) and since the fermions here can be considered massless, the e^+ particle must have a **right** helicity (e^+ are anti-fermions).

2. Denoting m the Z boson spin on the e^-e^+ axis, we get that for all productions $m = \lambda_{e^-} - \lambda_{e^+} = -1$ (reminder that λ_X is the helicity of particle X. We note m' the Z boson spin projection on the $\mu^-\mu^+$ axis, $m' = \lambda_{\mu^-} - \lambda_\mu^+$. The angular distributions of the different productions are given in Tab. 2 using the Wigner matrices formalism.

μ^- helicity	μ^+ helicity	m	m'	$\left\lvert d^j_{m'\,m}(\cos\theta)\right\rvert^2$
L	R	-1	-1	$\left(\frac{1+\cos\theta}{2}\right)^2$
R	L	-1	+1	$\left(\frac{1-\cos\theta}{2}\right)^2$
L	L	-1	0	N/A
R	R	-1	0	N/A

Table 2: Angular distribution of the different $\mu^+\mu^-$ production at SLC. The two last lines are forbidden due to the (axio-)vectorial couplings of the Z.

3. Therefore the total cross section is the sum of the two different production $\mu_L^-\mu_R^+$ and $\mu_R^-\mu_L^+$ weighted by their respective squared couplings $\left(-1/2 + 1\sin^2\theta_W\right)^2 = 0.073$ and $\left(0 + 1\sin^2\theta_W\right)^2 = 0.053$, thus following the respective fractions 0.58 : 0.42.

$$\sigma(e_L^- e^+ \to \mu^+\mu^-) \propto 0.58 \times \left(\frac{1+\cos\theta}{2}\right)^2 + 0.42 \times \left(\frac{1-\cos\theta}{2}\right)^2 \tag{8}$$

4. The angular distribution is not symmetric w/r to $\cos\theta$ (-1 and +1) and this asymmetry is due to the factors 0.58 and 0.42 which in turns relate to $\sin^2\theta_W$. Therefore by fitting the fractions to the angular distribution observed in data, one can measure $\sin^2\theta_W$.