Exercice IV

1. $B^0 \to \pi^+ \pi^-$

$m(B^0) \simeq 5279\ \text{MeV} > 2m(\pi^\pm) \simeq 280\ \text{MeV}$

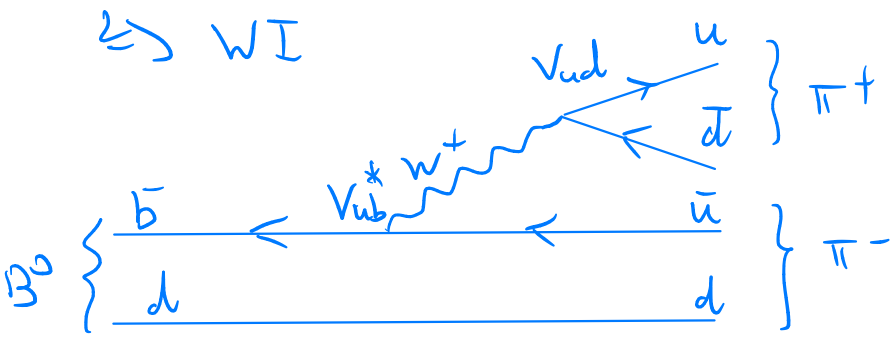
$Q_i = Q_f = 0$

L_l and $\mathcal{B}$ not involved

↳ process allowed

- $B_i = 1 \neq B_f = 0$
- $\tau(B^0) \sim ps$, typical for WI
- B^0 is the lightest beauty meson

↳ WI



$$\mathcal{M} \propto |V_{ub}^* V_{ud}|^2 \sim |\lambda^3 \cdot 1|^2 = \lambda^6$$

colour allowed tree diagram

2. $K^{*0}(892) \to K^+\pi^-$

$m(K^*) \simeq 892\,\text{MeV} > m(K^+) + m(\pi^-) \approx 500 + 140 = 640\,\text{MeV}$

$Q_i = Q_f = 0$

L_ℓ and $\mathcal{B}$ not involved

$\hookrightarrow$ allowed

- Only hadrons participate <u>and</u> no flavour violation ($S_i = S_f = +1$)
- $\Gamma(K^*) \simeq 46\,\text{MeV}$, typical for SI, <u>and</u> $K\pi$ is the dominant decay mode

$\Rightarrow$ SI

tree diagram

3. $J/\psi \to n\,\tau^+\tau^-\nu_\mu$

The decay is forbidden by:

- Energy conservation
  $m(J/\psi) \simeq 3100\,\text{MeV} < m(n) + 2m(\tau)$
  $\simeq 939 + 2 \cdot 1777 \simeq 4493\,\text{MeV}$
- $\mathcal{B}_i = 0 \neq \mathcal{B}_f = 1$
- $L_{\mu,i} = 0 \neq L_{\mu,f} = 1$

4. $\Lambda_b^0 \to \Lambda K^0 \bar{K}^0$

$m(\Lambda_b^0) \approx 5600\ MeV > m(\Lambda) + 2m(K^0)$

$\approx 1100 + 1000 = 2100\ MeV$

$Q_i = Q_f = 0$

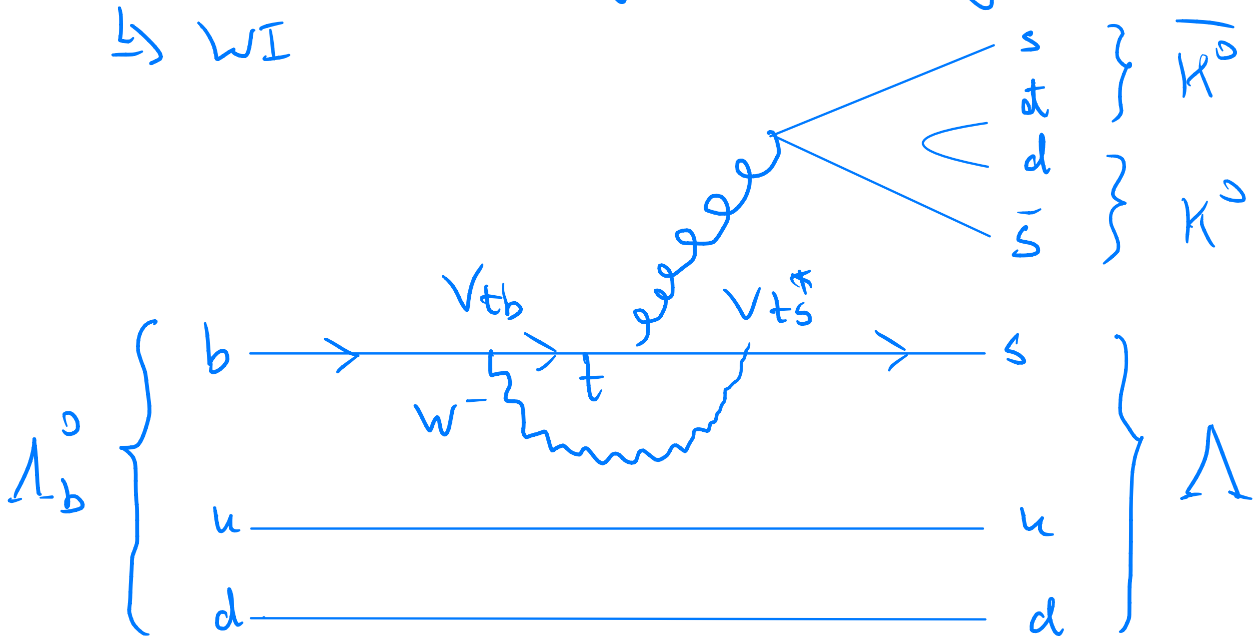
$\mathcal{B}_i = \mathcal{B}_f = 1$

L_ℓ are not involved

↳ allowed

- $B_i = -1 \neq B_f = 0$ (beauty not conserved)
- $\tau(\Lambda_b^0) \sim 1ps$, typical of WI
- Λ_b^0 is the lightest b-baryon

↳ WI



Gluonic penguin diagram

$M \propto |V_{tb} V_{ts}^*|^2 \sim |1 \cdot \lambda^2|^2 = \lambda^4$

5. $B^0_s \to \mu^+ \mu^- \mu^+ \mu^-$

$m(B^0_s) \simeq 5367\ \text{MeV} > 4 m_\mu \approx 440\ \text{MeV}$

$Q_i = Q_f = 0$

$L_{\mu,i} = L_{\mu,f} = 0$

L_e, L_τ and $\mathcal{B}$ are not involved

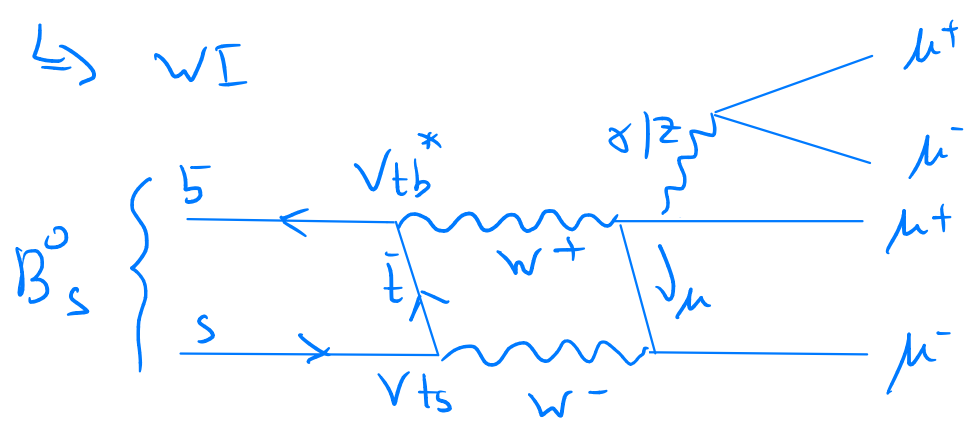
$\Rightarrow$ allowed

- $B_i = 1 \neq B_f = 0$
- $S_i = -1 \neq S_f = 0$

(Beauty and strangeness violation)

- $\tau(B^0_s) \sim 1\ \text{ps}$, typical of WI
- B^0_s is the lightest $\bar{b}s$ state

$\hookrightarrow$ WI



box diagram

$$\mathcal{M} \propto |V_{tb}^* V_{ts}|^2 \propto |1 \cdot \lambda^2|^2 = \lambda^4$$

Exercice V

Part 1

(a) In the CM:

$$p_B = (m_B, 0)$$

$$p_h = (E, p_h) \qquad (h = \rho \text{ or } K^*)$$

$$p_\gamma = (p_h, -p_h)$$

$$p_h^2 = (p_B - p_\gamma)^2$$

$$m_h^2 = (m_B - p_h)^2 - p_h^2 = m_B^2 - 2 m_B p_h$$

$$p_h = \frac{m_B^2 - m_h^2}{2 m_B}$$

Numerical values:

$$p_K = 2565 \text{ MeV}$$

$$p_\rho = 2583 \text{ MeV}$$

(b) In this 2-body decay,

$$d\Gamma = \frac{1}{32\pi^2} |M|^2 \frac{p_h}{m_B^2} d\Omega$$

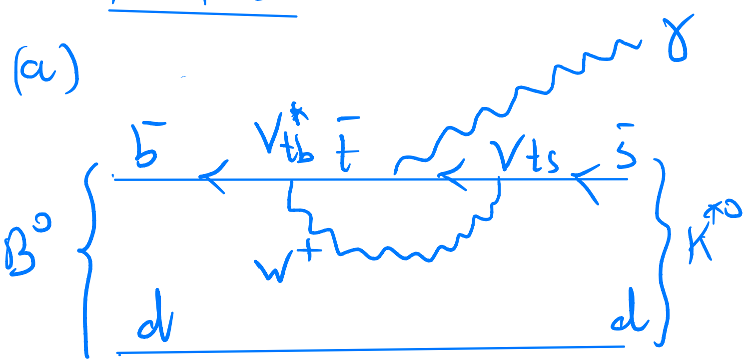
$\Downarrow$ Ratio of the PS factors:

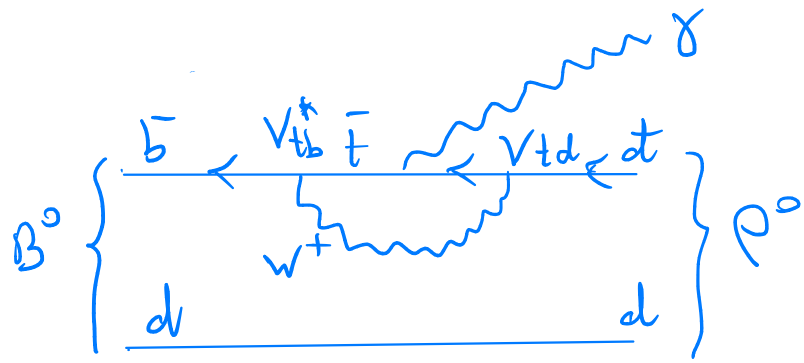
$$\frac{PS(B^0 \to K^* \gamma)}{PS(B^0 \to \rho^0 \gamma)} = \frac{p_K}{p_\rho} \simeq 0.993$$

The mass difference between the K^* and the ρ is small compared to m_B, so that the PS difference is negligible.

Part 2

(a)





(b) These are the lowest-order standard-model diagrams contributing to the processes. Other diagrams have more vertices and more W propagators. This also applies to the $u\bar{u}$ component of the ρ^0

(c) The topologies of the diagrams are the same, the phase-space factors are different by less than 1% and the diagrams above are dominant. Thus, the ratio of BF is expected to mainly come from CKM matrix elements:

$$R_{th} = \frac{BF(B^0 \to K^*\gamma)}{BF(B^0 \to \rho^0\gamma)} \simeq \frac{|V_{tb}^* V_{ts}|^2}{|V_{tb}^* V_{td}|^2} = \frac{|V_{ts}|^2}{|V_{td}|^2} \simeq$$

$$\simeq \frac{|-A\lambda^2|^2}{|A\lambda^3(1-\rho-i\eta)|^2} = \frac{1}{\lambda^2[(1-\rho)^2+\eta^2]} \simeq 22.5$$

The ratio of measured BFs from the PDG:

$$R_{exp} \simeq \frac{4.18 \times 10^{-5}}{8.6 \times 10^{-7}} \simeq 48.6$$

As expected, the estimation is rather good (off only by a factor 2)!

Differences occure due to hadronic corrections (form factors) and contributions from other diagrams, which may be different in the two processes.

Remark: error propagation gives an uncertainty of ~23% on the experimental ratio.

Part 3

(a) In general, modes with large BFs and charged "stable" particles:

$K^{*0} \to K^- \pi^+$

$\rho^0 \to \pi^+ \pi^-$

(b) Elements of the answer

- Associate tracks with opposite electric charges compatible with $K^\pm, \pi^\mp$ and a photon candidate.
- Consider invariant mass spectra of at least $K^- \pi^+$ and $K^- \pi^+ \gamma$.
- Signal events peak around the:
  K^{*0} mass in the first spectrum
  B^0 mass in the second spectrum.
- Other spectra may be used to study backgrounds.
- In the spectra, background events (e.g. combinatorial) are present and should be modelled.

(c) Both decays are due to SI (see ex. IV) thus the isospin formalism can be used. In addition, only half of the K^0 are observed as K^0_S, which implies a factor of $1/2$. (As $|K^0\rangle \simeq \frac{1}{\sqrt{2}}(|K^0_S\rangle + |K^0_L\rangle)$)

The isospin kets:

$|K^{*0}\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$ ($\bar{s}d$ state)

$|K^{+}\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$ ($\bar{s}u$ state)

$|K^{0}\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$ ($\bar{s}d$ state)

$$\left.\begin{aligned} |\pi^{-}\rangle &= |1\ -1\rangle \\ |\pi^{0}\rangle &= |1\ 0\rangle \end{aligned}\right\} \text{ members of the } \pi \text{ triplet}$$

$$|K^{+}\pi^{-}\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$|K^{0}\pi^{0}\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

Thus:

$$\frac{\Gamma(K^{*0}\to K^{+}\pi^{-})}{\Gamma(K^{*0}\to K_S^{0}\pi^{0})} = \frac{\Gamma(K^{*0}\to K^{+}\pi^{-})}{\frac{1}{2}\Gamma(K^{*0}\to K^{0}\pi^{0})} \simeq$$

(neglecting the PS-factor difference)

$$\simeq 2\cdot\frac{|\langle K^{*0}|H_s|K^{+}\pi^{-}\rangle|^2}{|\langle K^{*0}|H_s|K^{0}\pi^{0}\rangle|^2} \simeq 2\cdot\frac{\frac{1}{3}f_{1/2}}{\frac{2}{3}f_{1/2}} = 1$$

Part 4

- For a single particle, eigenstate of both $\hat{C}$ & $\hat{P}$ we just take the product $C \cdot P$. In this case there are no other contributions, in particular from spherical harmonics.
- The K^0_S is not a $\hat{C}$ eigenstate, but it is approximately a $\hat{C}\hat{P}$ eigenstate with $CP = +1$ (from its decay to $\pi^+\pi^-$)
- In the $K^0_S \pi^0$ final state: additional factor $(-1)^\ell$, where ℓ is the relative angular momentum between the final-state particles, to account for the spherical-harmonics contribution to P. The same applies for $\rho^0 \gamma$.

$$J^{PC}(\rho^0) = 1^{--}$$
$$J^P(K^{*0}) = 1^-$$
$$J(K^0_S) = 0 \quad ; \; CP(K^0_S) = +1$$
$$J^{CP}(\gamma) = 1^{--}$$
$$J(B^0) = 0$$
$$J^{PC}(\pi^0) = 0^{-+}$$

(a) $CP(\rho^0) = C(\rho^0)\,P(\rho^0) = (-1)(-1) = \underline{\underline{+1}}$

(b) $J_i = J(K^{*0}) = 1 \quad ; \quad \vec{J}_f = \underbrace{\vec{J}(K_s^0) + \vec{J}(\pi^0)}_{\vec{0}} + \vec{L}$

$J_f = J_i \Rightarrow \underline{l = 1}$

$CP(\underbrace{K_s^0\pi^0}) = CP(K_s^0)\cdot C(\pi^0)\,P(\pi^0)\,(-1)^l = \underline{\underline{+1}}$

in a decay of a $J=1$ particle

(c) $J_i = J(B^0) = 0 \quad ; \quad \vec{J}_f = \underbrace{\vec{J}(\rho) + \vec{J}(\gamma)}_{=\vec{J}(\rho\gamma)} + \vec{L}$

$J(\rho\gamma) = 0, 1$ or 2

$J_i = J_f \Rightarrow l = J(\rho\gamma)$

$$CP(\rho^0\gamma) = CP(\rho^0)\,\underbrace{CP(\gamma)}_{C(\gamma)\,P(\gamma)}\,(-1)^l = \begin{cases} +1 & l = 0, 2 \\ -1 & l = 1 \end{cases}$$

d) Similar answer. Like the ρ^0, K^{*0} is a vector particle, thus leaving the same freedom to l. In this case there is an additional $l_{K_s\pi^0} = 1$.

Part 5

(a) $b \to s\gamma$

$$J(b) = J(s) = \frac{1}{2} \quad ; \quad J(\gamma) = 1$$

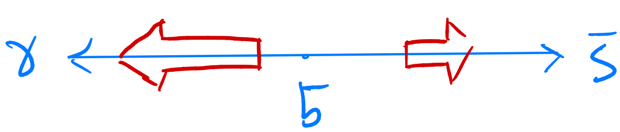
This is clearly a WI process (flavour violation, diagram in part 3...)

$\Rightarrow$ only left handed s-quarks interact, which applies for helicity of the s as we neglect its mass.

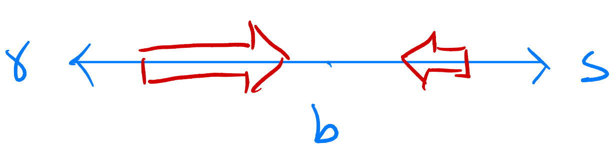
The γ has only two helicity states (± 1).

Thus, given that the initial b-quark has $J = \frac{1}{2}$, the photon polarisation must be the same as that of the s-quark (otherwise the total angular-momentum projection on the thrust axis is $3/2$).

We conclude that:



$\bar{b} \to \bar{s}\gamma \iff$ RH γ only



$b \to s\gamma \iff$ LH γ only

(b) Due to the non-zero mass of the final-state quark, there is a contribution from the opposite-helicity state of a given chirality. (This component is smaller for the d quark)

(c) The 2 decays, $B^0 \to \rho^0 \gamma$ and $\bar{B}^0 \to \rho^0 \gamma$ are allowed. (This is clear, e.g., from the fact that the ρ^0 is created as a $d\bar{d}$ state).

Thus neglecting any subtelties, we expect CP violation of type 3 (in interference between mixing and decay).

The direct CPV is expected to be negligible as there is only one dominant diagram.

(d) Here, unlike for $\rho^0\gamma$, $K^{*0}\gamma$ are not always accessible with both B^0 and $\bar{B}^0$ decays, as $B^0 \to K^{*0}\gamma$ ($\bar{b} \to \bar{s}\gamma$ transition) and $\bar{B}^0 \to \bar{K}^{*}\gamma$ ($b \to s\gamma$ ").

On the other hand, both K^{*0} and $\bar{K}^{*0}$ decay into $K^0_S \pi^0$. Thus $f = K^0_S \pi^0$.

(e) When considering the γ handedness (γ_L & γ_R) we have shown that the dominant decays are

$$\bar{B}^0 \to \rho^0 \gamma_L$$

$$B^0 \to \rho^0 \gamma_R$$

The final states from B^0 and $\bar{B}^0$ decays differ and therefore there is no interference between the direct decay amplitude, and that of the mixing and decay.
The expected CPV is thus 0, up to small corrections due to the d quark mass and contributions from higher order diagrams. (Any significant deviation from this would mean that a BSM amplitude contributes).