

g.

$$p \rightarrow 1 \ 2 \ 3$$

$$(M) \quad m_i$$

$$m_{23}^2 = (p_2 + p_3)^2 =$$

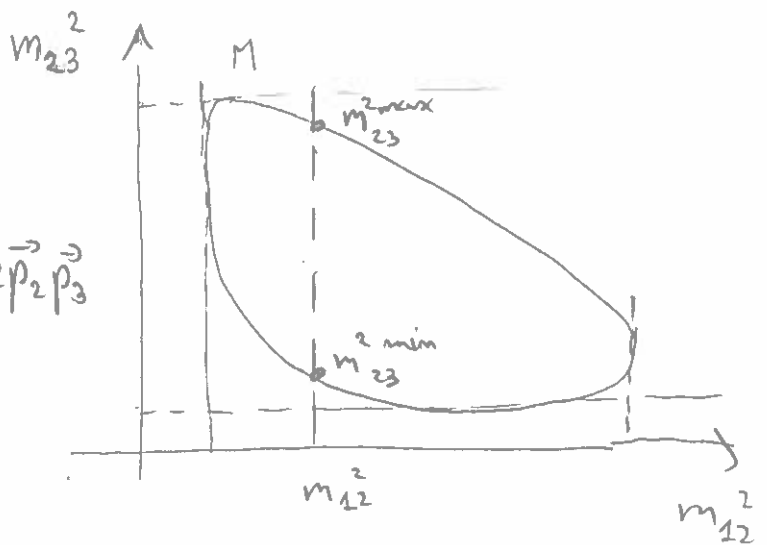
$$= m_2^2 + m_3^2 + 2E_2E_3 - 2\vec{p}_2\vec{p}_3$$

See:

PDG 2016: p. 324

PDG 2018: p. 239

PDG 2020: p. 252



To understand the kinematic boundaries:
 e.g. along the m_{23}^2 axis, think of the system of particles 2 and 3 as if they were a single particle of mass m_{23} .

⊛ Min value: $m_2 + m_3$

Possible kinematically? What does it correspond to?

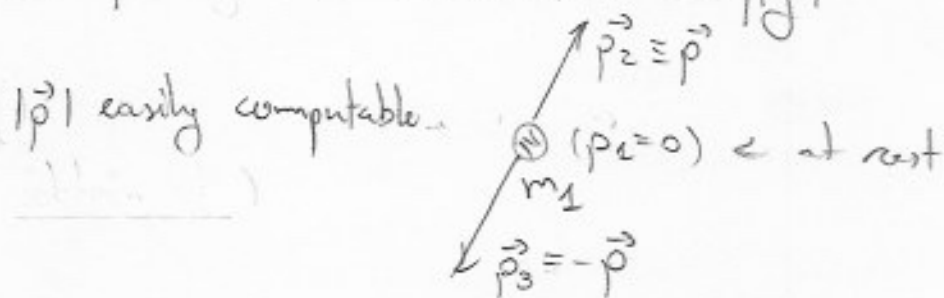
Yes! The 2 particles move together (same velocity) as if they were a single object.

It is clear that then $m_{23}^2 = (m_2 + m_3)^2$
 (think about the rest frame of m_2/m_3 .)

⊛ Max value: $M - m_1$ (in the rest frame of particle 1)
 Same questions ...

Yes! In the rest frame of M particle 1 and the syst 23 are produced "at rest" (all the difference $M - m_1$ becomes m_{23})

In fact, the kinematic config:



$$P_2 = (E_2, \vec{p})$$

$$P_3 = (E_3, -\vec{p})$$

$$P_{tot} = (M - m_1, 0)$$

$$(P_{tot} - P_3)^2 = \frac{P_2^2}{m_2^2}$$

$$P_{tot}^2 - 2P_{tot} \cdot P_3 + P_3^2 = P_2^2$$

$$(M - m_1)^2 + m_3^2 - m_2^2 = 2(M - m_1)E_3$$

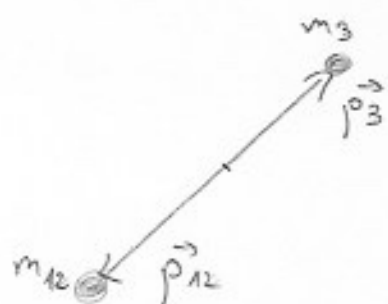
$$E_3 = \frac{\dots}{2(M - m_1)}$$

$$p = \sqrt{E_3^2 - m_3^2} = \dots$$

Contours of the kinematic boundaries:

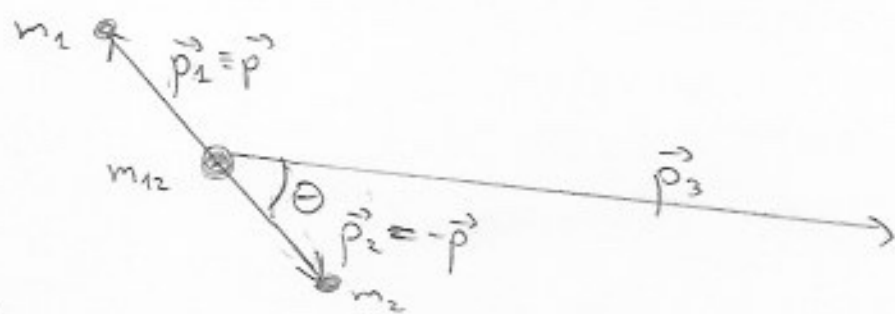
at a given m_{12}^2 : $[m_{23}^{\min}, m_{23}^{\max}]$?

M rest frame:



Only thing that can change m_{23} : angle between 2 and 3.

In the rest frame of 12:



Modules of all momenta are easily computable
 $\Theta \equiv$ helicity angle

It is clear that:

$$m_{23}^{\min} \Leftrightarrow \Theta = 0$$

$$m_{23}^{\max} \Leftrightarrow \Theta = \pi$$

The computation itself does not have any conceptual difficulty.