Exercise sheet № 7 - Weak interaction - basics

Unless mentioned otherwise, use the PDG booklet to find the necessary particle properties, experimental measurements and physical constants.

Exercise 1

The reaction $^{14}O \rightarrow {}^{14}N^{*}\, e^{+}\, \nu_e$ is a nuclear β^{+} decay. It is the unique decay mode of the isotope ^{14}O. The corresponding decay width is:

$$\Gamma_n = \frac{G_n^2 E_0^5}{30\pi^3},$$

where E_0 is the energy released during the reaction.

a) Verify the dimensional homogeneity of this expression.

b) We give E_0=1.81 MeV and the radioactive half-life period of the ^{14}O, $T_{1/2}$ =70.64 s. Find the numerical value of G_n.

c) The width of the μ decay to an electron and neutrinos ($\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$) is

$$\Gamma_\mu = \frac{G_\mu^2 m_\mu^5}{192\pi^3}.$$

Using τ_μ=2.2 10^{-6} s, find the numerical value of G_μ.

d) Compare G_n et G_μ, and comment.

Exercise 2

We consider the decay of D^+ and D^0 mesons into final states with one positron:

(i). $D^+ \rightarrow e^+X$ (ii). $D^0 \rightarrow e^+X$

Using the lifetimes of the D mesons and the branching fractions of the two processes[1] compute the value of $\Gamma(D^+ \rightarrow e^+X)/\Gamma(D^0 \rightarrow e^+X)$. Comment the result.

Exercise 3

Using Feynman diagrams, explain why the reaction $\bar{\nu}_\mu e^- \rightarrow e^- \bar{\nu}_\mu$ gives unique evidence of the existence of neutral currents (Z boson exchange), unlike the reactions $\bar{\nu}_e e^+ \rightarrow \bar{\nu}_e e^+$ et $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$.

[1] A branching fraction of this kind is called "inclusive", in contrast to an "exclusive" branching fraction, where all the final state particles are specified.

Exercise 4

We remind that the D^0 meson is a bound cu state with a mass of 1865 MeV.

a) We are first interested in the decay of the D^0, considering that only the heavier quark participates in the reaction. Draw the Feynman diagrams of the dominant transitions with respect to the CKM-matrix elements involved.

b) Neglecting other processes, deduce a rough estimation of the branching fraction of the semileptonic decay $D^0 \to \mu^+ \nu_\mu X$, where X represents a system of hadrons. To answer, suppose that the phase space is roughly similar for all the processes in a) above (justify this assumption). Compare your estimated branching fraction with that quoted in the PDG. Comment.

c) Repeat the exercise with the B^0 meson (bd) and estimate the semileptonic branching fraction $BF(B^0 \to \mu^+ \nu_\mu X)$.

d) Like the muon, the b-quark inside the B meson decays via weak interaction. Use the analogy between these two cases to estimate the lifetime of the B^0. For each of these cases, draw the Feynman diagrams of the dominant processes. We remind that, following Fermi's approximation, the width of the muon is of the form $\Gamma_\mu = K m_\mu^5$. We will use the values $m_b = 4.5$ GeV, $V_{cb} = 0.04$, $m_\mu = 105.7$ MeV and $\tau_\mu = 2.2\ 10^{-6}$ s.

e) How can you explain the fact that the B^0 has a longer lifetime of the D^0 despite the fact that $m_B > m_D$?

Exercise 5

Write explicitly a few semileptonic decay processes of a top quark in a u-quark (possibly as a "cascade" of a few consecutive decays). Find the three processes whose contribution to the amplitude is maximal. Draw the corresponding Feynman diagrams, associating to each adequate vertex a CKM-matrix element. Give the approximate ratio of the rates of the three dominant processes.

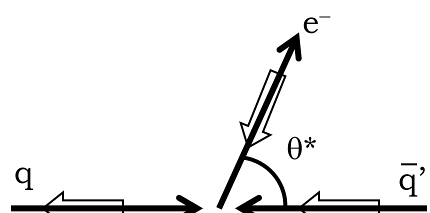
Exercise 6

The W boson is produced in high-energy $p\bar{p}$ collision following the process $p\bar{p} \to W^- X$. It is detected via the decay $W^- \to e^- \bar{\nu}_e$.

a) The leptonic decay branching fraction $BF(W \to \ell\ \nu) \approx 10\%$, independently of the flavour of the lepton ℓ. If the W had a 0-spin, would the branching fractions of its decay to a muon and an electron be the same? To answer, use arguments related to kinematics and phase space.

b) Draw one of the Feynman diagrams of the process $p\bar{p} \to W^- X$ and another one for $p\bar{p} \to ZX$. These two processes allowed the discovery of the W and Z bosons in CERN in the 80s.

c) Justify the fact that the only allowed helicity configuration for the interaction $qq \to e^- \bar{\nu}_e$, in the limit $m_q = m_{\bar{q}} = 0$ is the following:



d) The angle θ^* is defined between the direction of the outgoing electron in the centre of mass of the W boson and the flight direction of the W in the laboratory frame. The latter also defines the positive sense of the z-axis. We suppose that the W boson has spin 1. Deduce the angular distribution of the electron in the centre of mass of the W ($d\sigma/d\cos\theta^*$). Use the expression of the Wigner matrix elements given in the PDG, the additional notes and the figure above. We do not require the full expression of the differential cross section, but only its angular dependence $f(\theta^*)$ ($d\sigma/d\cos\theta^* \propto f(\theta^*)$).

e) Same question, supposing that the spin of the W is 2. Suggest a method to determine which one of the two W-spin hypotheses is correct.

Exercise 7

We now examine the reaction $e^+e^- \rightarrow \mu^+\mu^-$. We specify that the centre-of-mass energy is large enough for the muons to be ultrarelativistic, and small enough so that the reaction is mediated predominantly by a photon (J=1).

a) What can you say about the helicities of the interacting particles? What are the possible projections of the total angular momenta in the initial and final states on the axes defined by the directions of the negatively charged particles?

b) Draw the four schemes, describing the possible configurations of momenta and helicities of the incoming and outgoing particles in the centre-of-mass frame. The angle θ is defined between the directions of the incoming e^- and the outgoing μ^- in this frame.

c) Using the Wigner matrix elements $d^J_{\lambda_1\lambda_2}(\theta)$, give the θ–dependence of the amplitudes corresponding to each of the schemes drawn in a) above.

d) Compare your results to Fig. 1 below. We indicate the helicity amplitudes are the same for all the four helicity configurations.

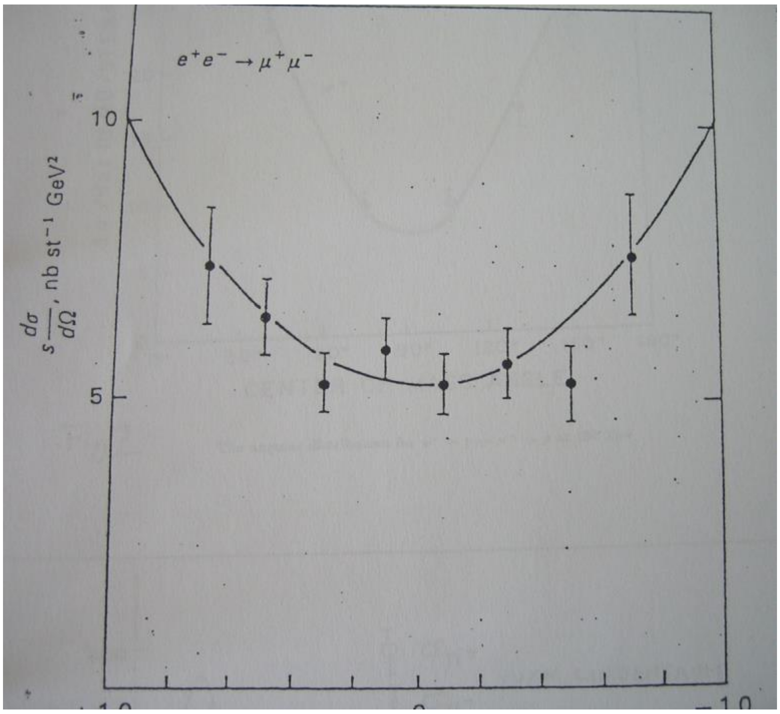


Figure 1: Measured $e^+e^- \to \mu^+\mu^-$ cross section as a function of $\cos(\theta)$

<u>Exercise 8</u>

We now examine the reaction $\pi^+ p \rightarrow \Delta^{++} \rightarrow \pi^+ p$

a) Draw the four schemes describing the possible configurations of momenta and helicities of incoming and outgoing particles in the centre-of-mass frame. Considering parity transformation, show that there are only two independent helicity amplitudes.

b) We suppose that, for these two amplitudes, the factors that do not contain any angular dependence (helicity amplitudes) are the same. Write the angular dependence of the differential cross section, in terms of Wigner matrix elements, for a Δ^{++} resonance of a given spin J. We note as θ^* the angle between the z-axis and the z'-axis, which are oriented in the direction of the incoming pion and the outgoing proton, respectively, in the centre of mass frame.

c) Deduce the angular distribution of the outgoing protons in the centre of mass, in the cases J=1/2 and J=3/2. Compare the result to Fig. 2 below. Does this result allow determining the spin of the Δ resonance, in case it is a-priori unknown?

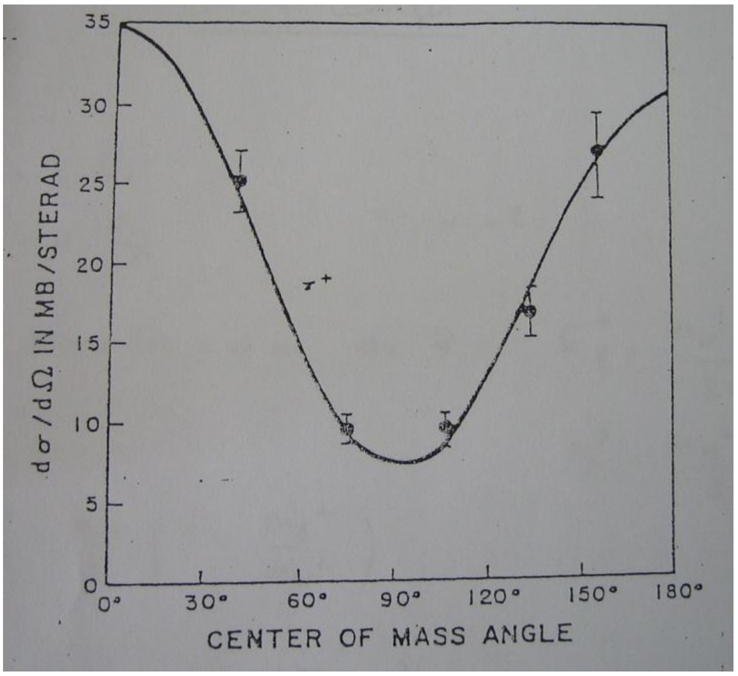


Figure 2: The π^+p cross section as a function of the centre of mass angle

d) Using the additional notes, give the expression of the maximal cross section for the process $\pi^+p \to \Delta^{++} \to \pi^+p$. Compute it numerically for different values of J and compare to the maxima of the different distributions in Fig. 3. Show that this result allows extracting the same spin of the Δ^{++} resonance as above.

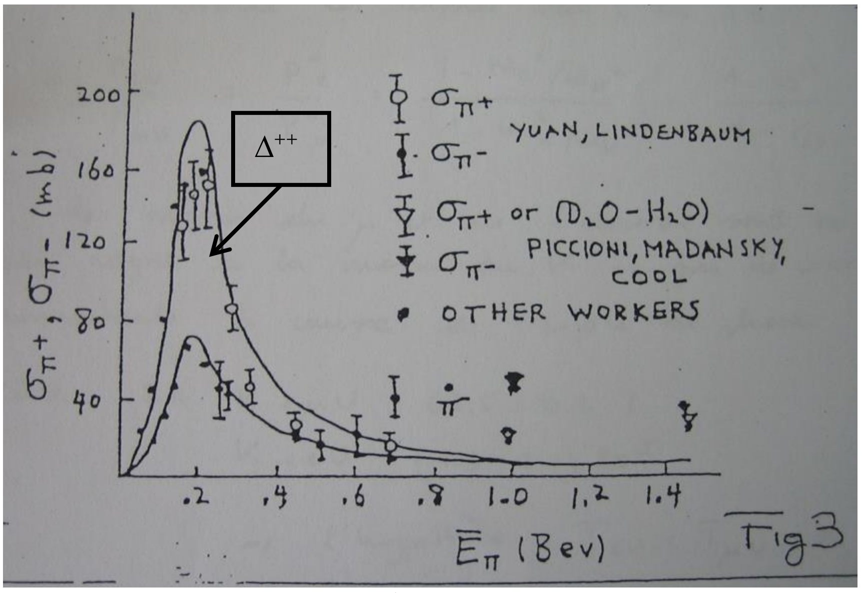


Figure 3