# Midterm exam of Particle Physics
## Tuesday November 14th 2023

*Duration: 3 hours*
*6 printed pages*

*Allowed material: PDG booklet, simple calculator.*
**Solve on two separate sheets exercises I-II and exercises III-IV.**

| *Approximate duration per exercise:* | | | |
|---|---|---|---|
| Ex. I: | 15 min. | Ex. II: | 75 min. |
| Ex. III: | 25 min. | Ex. IV: | 65 min. |

## Exercise I
*Questions on the lectures*

Give short and succinct answers to the questions below. The best possible answer is the shortest one that details comprehensively all the relevant arguments.

1. Briefly describe the historical context of the proposal of the quark model, along with relevant dates. Explain the logic on which the proposal was based.
2. What is the relation between the intrinsic width and the lifetime of a particle? What is the reason for this relation? Remind and explain the definition of the partial width and the logic behind it.
3. Explain the evolution of the fine structure constant $\alpha$ with energy involved in the reaction.

## Exercise II
Production of axion-like particles (ALPs)

**The different parts of this exercise are mostly independent.**

We consider, at the LHC, the production of the Higgs boson ($h$) and its decay to a pair of ALP particles ($a$). Each of the $a$ particles subsequently decays to a pair of photons ($a \to \gamma\gamma$). Therefore, we study the process $pp \to h + X$ ; $h \to aa \to (\gamma\gamma)(\gamma\gamma)$. Here $X$ represents other particles produced along with the Higgs boson. We will not consider them in the following.

1. **Particles production vs rapidity**
   At a hadron collider, one can show that the number of particles produced per unit of rapidity, $y$, is mostly constant for a large range of rapidities. This phenomenon, known as the rapidity plateau, means that $\frac{dN}{dy d\varphi} = \text{cst}$, with $\varphi$ the azimuthal angle in spherical coordinates.

   We consider massless particles and we remind you that the definition of the pseudo-rapidity $\eta \equiv -\ln \tan \frac{\theta}{2}$ where $\theta$ is the polar angle in the spherical coordinate system.

   (a) Express $\cosh \eta$ and $\sinh \eta$ as a function of $\theta$

(b) We consider the pseudo-rapidity difference $\delta\eta \equiv \eta' - \eta$ where $\eta'$ corresponds to the polar angle $\theta' \equiv \theta + \delta\theta$, and we assume $\delta\theta \ll 1$. Compute $\delta\eta$ as a function of $\delta\theta$ to first order in $\delta\theta$.

(c) Express the differential solid angle $\delta\Omega \equiv \delta\cos\theta\delta\varphi$ as a function of $\delta\eta\delta\varphi$, $\cosh\eta$ and $\sinh\eta$.

(d) Compute the particle-production-rate ($\frac{dN}{d\Omega}$) dependence on $\eta$. Is it also constant? Draw a sketch of $\frac{dN}{d\Omega}$ as a function of $\cosh\eta$.

(e) Imagine a spherical detector composed of multiple channels paving the sphere and pointing to the centre of the detector. The detection area (front face of a single channel) has a surface of $2.5 \times 2.5$ cm$^2$. The radius of the sphere is 2 m. What is the typical solid angle $\delta\Omega_{\mathrm{ch}}$ of a single channel?

(f) Given the responses to the 2 previous questions, how does the irradiation dose, received per channel as a function of the channel $\eta$, evolve? Find some problematic consequences for the detector at high rapidity (no computation required).

2. **Kinematics of $h \to aa \to 4\gamma$ decay**
We consider that the Higgs boson is produced at rest at the centre of the detector.

(a) What are the energies $E_a$ and tri-momenta $|\vec{p}_a|$ of both ALP particles, expressed as a function of $m_h$ and $m_a$?

(b) We consider the decay of one of the $a$ particles $a \to \gamma\gamma$. Express its mass, $m_a$, as a function of $E_{\gamma 1}$, $E_{\gamma 2}$ and $\Delta\theta_{\gamma\gamma}$, respectively: the energies of photon 1, photon 2 and the angular separation between the two photons.

(c) Express the energy $E_a$ of the particle $a$ as a function of its Lorentz boost $\gamma_a$ and its mass $m_a$ (if you don't know the formula, you can find it by considering a Lorentz boost from the particle rest frame to the laboratory frame). Deduce the value of the Lorentz factor $\gamma_a$ as a function of $m_h$ and $m_a$. What is the value of the velocity $\beta_a$?

(d) We assume $E_{\gamma_1} = E_{\gamma 2} \equiv E_\gamma$, what is the value of $E_\gamma$ as a function of $m_h$?

(e) Assuming $m_a \ll m_h$, demonstrate that $\Delta\theta_{\gamma\gamma} \ll 1$ and express $\Delta\theta$ as a function of $m_h$ and $m_a$ (use a first-order approximation of the mass expression from question II.2.b).

(f) Assuming $m_a = 0.1$ GeV, what is the value of $\Delta\theta_{\gamma\gamma}$ in radian. Compared to the angular size of one channel of the detector described in question II.1.f, are these 2 photons easy to distinguish from an experimental point of view?

3. **Study of the decay $a \to \gamma\gamma$**
The required Feynman rules are given in the appendix, $\alpha$ represents the fine structure constant and $\frac{c_{\gamma\gamma}}{\Lambda}$ the dimensional coupling strength of the $a$ particle to photons. For numerical applications, we will use $\frac{c_{\gamma\gamma}}{\Lambda} = 1$ TeV$^{-1}$.

We note the 4-momenta: $p_a$ for $a$, $k_1$ and $k_2$ for the 2 photons.

(a) Give the Feynman diagram describing the decay at LO. Express the corresponding matrix element $\mathcal{M}_{a\gamma\gamma}$.

(b) The matrix element is given by $|\mathcal{M}_{a\gamma\gamma}|^2 = 16\,[(k_1.k_2)^2 - k_1^2\,k_2^2]\,|4\pi\alpha\frac{c_{\gamma\gamma}}{\Lambda}|^2$. Express $|\mathcal{M}_{a\gamma\gamma}|^2$ as a function of $m_a$, $\frac{c_{\gamma\gamma}}{\Lambda}$ and $\alpha$.

(c) Using the master formula for $d\Gamma$ (you can find it in the PDG), re-demonstrate that for this 2-body decay:

$$\frac{d\Gamma}{d\Omega} = \frac{1}{4(4\pi)^2 m_a}\,\frac{2k^*}{m_a}\,|\mathcal{M}_{a\gamma\gamma}|^2$$

with $k^* \equiv |\vec{k}_\gamma|$ the norm of the 3-momentum of the photon in the centre of mass of the particle $a$.

(d) Using the 2 previous questions, express $\Gamma_{\gamma\gamma} \equiv \Gamma\left(a \to \gamma\gamma\right)$ as a function of $m_a$, $\alpha$ and $\frac{c_{\gamma\gamma}}{\Lambda}$.

(e) Express the typical distance $L_a$ travelled by the $a$ particle as a function of $\gamma_a$, $\beta_a$ and $\tau_a$, which are, respectively, the Lorentz factor, the velocity and the lifetime of the $a$ particle. Express $L_a$ as a function of $\gamma_a$, $\beta_a$ and $\Gamma_a$, with $\Gamma_a$ the total width of the $a$ particle.

(f) The length of the detector is $L_{\text{det}} = 2$ m. What is the numerical value of $L_{\text{det}}$ in GeV$^{-1}$ unit?

(g) What must be the relation between $L_a$ and $L_{\text{det}}$ so that the particle $a$ decay products can be detected?

(h) We assume that $\gamma_a\beta_a \approx \frac{m_h}{2\ m_a}$ and $\mathcal{B}(a \to \gamma\gamma) = 1$. Express $\Gamma_a$ as a function of $\Gamma_{\gamma\gamma}$? What is the minimal mass of $a$ (noted $m_a^{\text{det}}$) so that the particle $a$ decay products can be detected (including the numerical computation)?

# Change sheets here

## Exercise III

*Allowed and forbidden processes, Feynman diagrams*

For each of the processes below, determine whether it is allowed or strictly forbidden in the standard model. For the forbidden processes, explain why they are forbidden, giving *all the possible reasons* (here we do not require to take into account multiplicative quantum numbers, angular momentum or isospin). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Note on the diagram the names of all real and virtual particles.

1. $K^- \longrightarrow \pi^-\pi^0$
2. $J/\psi \longrightarrow e^+e^-$
3. $\Lambda \longrightarrow n\tau^-\nu_\tau$
4. $p\pi^- \longrightarrow \Lambda K^0$
5. $\overline{D}^0 \longrightarrow \mu^+\mu^-$

## Exercise IV

*Exotic hadron decaying into $\eta_c\pi^-$*

**The different parts of this exercise are mostly independent.**

Several exotic hadrons containing a heavy quark-antiquark pair have been observed in recent years. Their internal structure, in terms of quark content, as well as their binding mechanisms, are still to be clarified, for instance by studying their quantum numbers. Some of them may be tetraquarks (bound states composed of two quarks and two antiquarks), or bound states of quarks and valence gluons.

In this exercise, we will look at several aspects of a search, performed by the LHCb collaboration, for such an exotic hadron with a mass around 4000 MeV, decaying into $\eta_c\pi^-$. This presumed hadron is not yet in the PDG and will be noted as $X^-$ in the following. The results of the search have been published by the LHCb collaboration in 2018 [Eur. Phys. J. C **78** (2018) no.12, 1019]. LHCb searched for this decay by analyzing $B^0 \to K^+\eta_c\pi^-$ decays. $\eta_c$ is the lightest $c\bar{c}$ meson, also denoted $\eta_c(1S)$.

1. **Experimental considerations**

   (a) LHCb chose to reconstruct the $\eta_c$ via its decay into $p\overline{p}$. Looking at the list of the $\eta_c$ decay modes in the PDG, briefly explain what are the experimental advantages of $p\overline{p}$ decay mode.

   (b) The figure below shows the $p\overline{p}$ invariant-mass spectrum, in which most of the backgrounds have been subtracted. Comment on the two peaks present in the spectrum in light of relevant properties of the $J/\psi$ and $\eta_c$ (see PDG), and in particular their widths.

   (c) Which other invariant-mass spectra can you assume that LHCb considered in this search? (Give at least the most relevant one, and if possible two or three relevant spectra.)

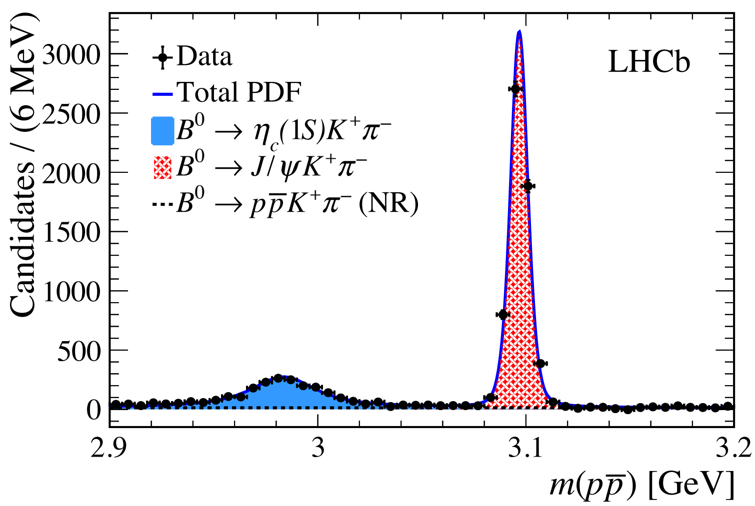



Figure 1: Distribution of the $p\overline{p}$ invariant mass from the LHCb article [Eur. Phys. J. C **78** (2018) no.12, 1019]. Most of the backgrounds have been subtracted from this spectrum by an event-reweighting technique. The dots with error bars correspond to the data histogram. The full azure, tight-cross-hatched red and dashed-black line areas show the $\eta_c$, $J/\psi$ and non-resonant $p\overline{p}$ contributions.

2. **Quantum numbers of the $X^-$ hadron**
   As a reminder, we are considering $X^- \to \eta_c\pi^-$ decays.

   (a) Without any assumption on the interaction at play, what can you tell about the different charges (additive quantum numbers) of the hadron $X^-$? Express the spin of $X^-$ as a function of $\ell$, the relative orbital angular momentum between the $\eta_c$ and the $\pi^-$ mesons.

   From now on we will take into account that the decay is due to the strong interaction.

   (b) Determine other charges of the exotic hadron $X$ and its isospin ($I$ and $I_3$).

   (c) Exploiting the relevant conservation laws, find its possible values of spin-parity, $J^P$. Consider all possible $\ell \leq 2$.

   (d) What can you tell about the charge conjugation $C(X^-)$ of the particle $X^-$?

   (e) Using simple arguments, determine what is the most probable quark content of the exotic hadron if it is assumed to be a tetraquark.

   (f) What is the $G$-parity of this particle? Along with an explanation, give at least two final states $F$, composed of a $c\overline{c}$ meson and a light meson, for which the transition $X^- \to F$ is forbidden by strong interaction.

3. **Aspects of the $\eta_c$ decays**

(a) Under the assumption that the $\eta_c$ meson decays into $p\bar{p}$ via the strong interaction, give all the relevant arguments indicating that the $\eta_c \to \Sigma^+\bar{\Sigma}^-$ decay is also due to the strong interaction (note that the $\bar{\Sigma}^-$ is an antibaryon).

(b) Draw the Feynman diagrams of the two aforementioned strong-interaction decays.

(c) Write down the isospin kets ($|I, I_3\rangle$) of the five particles involved in these two decays. Using the approximation of a similar phase-space factor, estimate the ratio of the two partial widths. Briefly explain.

(d) Compare your estimate with the experimental measurements from the PDG and comment.

(e) What may be the reason for these two decays not to proceed dominantly via the strong interaction? (Hint: the colour, as all the other charges, is conserved at any vertex.) Which interaction would you then assume to compete with the strong interaction? Explain.

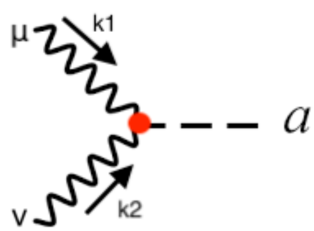
We now consider the decay $\eta_c \to K^+K^-\pi^0$.

(f) The relative orbital angular momentum between the two kaons is denoted $\ell_{12}$ and that between the centre of mass of the two kaons and the $\pi^0$ is denoted $\ell_3$. Express the parity, $P_f$, and the charge conjugation, $C_f$, of the final state as a function of $\ell_{12}$ and $\ell_3$. What is/are the possible value/values of the total angular momentum of the final state?

(g) Explain why it is allowed to use $P$ and $C$ conservation in this decay. Use the relevant conservation laws to set constraints on $\ell_{12}$ and $\ell_3$.

Figure 2: QED Feynman rules. $s$ refers to the (anti-)fermion spin and $\lambda$ to the photon helicities.

| | | | |
|---|---|---|---|
| Incoming fermion: |  | $\rightarrow$ | $u\ (\vec{p}, s)$ |
| Incoming antifermion: |  | $\rightarrow$ | $\overline{v}\ (\vec{p}, s)$ |
| Outgoing fermion: |  | $\rightarrow$ | $\overline{u}\ (\vec{p}, s)$ |
| Outgoing antifermion: |  | $\rightarrow$ | $v\ (p, s)$ |
| Incoming photon: | $\mu$  | $\rightarrow$ | $\epsilon_\mu(\vec{k}, \lambda)$ |
| Outgoing photon: |  $\mu$ | $\rightarrow$ | $\epsilon_\mu(\vec{k}, \lambda)^*$ |




$$V^{a\gamma\gamma}_{\mu\nu} \equiv 4\ i\ \epsilon_{\mu\nu\rho\sigma}\ k_1^\rho\, k_2^\sigma \left(4\pi\alpha \frac{c_{\gamma\gamma}}{\Lambda}\right)$$