Correction to exercises I + II of the 2024 midterm exam of Particle Physics

Exercise I

Questions on the lectures

Give short and succinct answers to the questions below. The best possible answer is the shortest one that details comprehensively all the relevant arguments.

1. Briefly describe the historical context of the proposal of the quark model, along with relevant dates. Explain the logic on which the proposal was based.

   Answer In the 1950s, novel experimental developments, mainly bubble chambers and particle accelerators, led to the discovery of many hadrons, without a convenient theory that was able to explain their properties. In 1961, Murray Gell-Mann and Yuval Ne'eman proposed a classification of these particles, that appeared as multiplets in the I_3, Y (isospin projection, hypercharge) plane. The structures were identified as multiplets of the SU(3) symmetry: two meson octets, one baryon octet, and 1 baryon decuplet, where particles in the same multiples have the same J^P quantum numbers. These structures indicated that hadrons are constituted of three underlying objects, called quarks (up, down and strange). Constructing numerous hadrons as bound states of fewer constituents helped to organize them, and is understood to be a consequence of the flavour-symmetry structure of the three lightest quarks.

2. What is the relation between the intrinsic width and the lifetime of a particle? What is the reason for this relation? Remind and explain the definition of the partial width and the logic behind it.

   Answer Any unstable particle with a lifetime of τ has an intrinsic width of $\Gamma = \hbar/\tau$ (in natural units). This relation is related to the uncertainty principle: given that the lifetime, which is distributed exponentially, has an uncertainty of τ, also the intrinsic energy of the particle, its mass, must have an uncertainty, noted Γ, with $\tau\Gamma = \hbar$. Furthermore, the Fourier transform of the exponential distribution of lifetime yields a Breit–Wigner distribution of the mass, with an FWHM of Γ. The partial width of the particle X decaying to the final state f is defined as $\Gamma_{X\to f} = \mathrm{BF}_{X\to f}\Gamma_X$. The total width is Γ_X proportional to the total probability of the particle X to decay (a short lifetime and thus a large width are related to a large probability). Multiplying Γ_X by the branching fraction $\mathrm{BF}_{X\to f}$ (the fraction of X undergoing the $X \to f$ decay, is thus clearly related to the probability of this specific decay.

3. Explain the evolution of the fine structure constant α with energy involved in the reaction.

   Answer The fine structure constant relates to the electric charge as $\alpha = e^2/(4\pi)$. We consider that the fine structure constant depends on the energy involved in the reaction because of quantum corrections to the vacuum polarisation, which are corrections to the photon propagator (loop diagrams, a diagram was appreciated). These corrections relates to virtual e^+e^- pair creation. The fine structure constant increases with energy from 1/137 at very low energy to 1/128 at the Z boson mass (this last number was not required).

Exercise II

Production of axion-like particles (ALPs)

The different parts of this exercise are mostly independent.

We consider, at the LHC, the production of the Higgs boson (h) and its decay to a pair of ALP particles (a) which are pseudo-scalar particles. Each of the a particles subsequently decays to a pair of photons ($a \to \gamma\gamma$). Therefore, we study the process $pp \to h + X$; $h \to aa \to (\gamma\gamma)(\gamma\gamma)$. Here X represents other particles produced along with the Higgs boson. We will not consider them in the following.

1. **Particles production vs rapidity**

   At a hadron collider, one can show that the number of particles produced per unit of rapidity, y, is mostly constant for a large range of rapidities. This phenomenon, known as the rapidity plateau, means that $\frac{dN}{dyd\varphi} = \text{cst}$, with φ the azimuthal angle in spherical coordinates.

   We consider massless particles and we remind you that the definition of the pseudo-rapidity $\eta \equiv -\ln\tan\frac{\theta}{2}$ where θ is the polar angle in the spherical coordinate system.

   (a) Express $\cosh\eta$ and $\sinh\eta$ as a function of θ

$$\begin{aligned}\cosh\eta &= \frac{e^{\eta}+e^{-\eta}}{2} = \frac{1}{2}\left(\frac{1}{\tan\theta/2} + \tan\theta/2\right)\\ &= \frac{\cos^2\theta/2 + \sin^2\theta/2}{2\,\cos\theta/2\,\sin\theta/2}\\ \cosh\eta &= \frac{1}{\sin\theta}\end{aligned} \tag{1}$$

$$\begin{aligned}\sinh\eta &= \frac{e^{\eta}-e^{-\eta}}{2} = \frac{1}{2}\left(\frac{1}{\tan\theta/2} - \tan\theta/2\right)\\ &= \frac{\cos^2\theta/2 - \sin^2\theta/2}{2\,\cos\theta/2\,\sin\theta/2}\\ \sinh\eta &= \tan^{-1}\theta\end{aligned} \tag{2}$$

   (b) We consider the pseudo-rapidity difference $\delta\eta \equiv \eta' - \eta$ where η' corresponds to the polar angle $\theta' \equiv \theta + \delta\theta$, and we assume $\delta\theta \ll 1$. Compute $\delta\eta$ as a function of $\delta\theta$ to first order in $\delta\theta$.

$$\delta\eta = \eta' - \eta = -\ln\frac{\tan\frac{\theta+\delta\theta}{2}}{\tan\frac{\theta}{2}} \tag{3}$$

   Let's first concentrate on developing to order one $\tan(\alpha+\delta\alpha)$ with $\delta\alpha \ll 1$.

$$\begin{aligned}\tan\alpha+\delta\alpha &= \frac{\cos\alpha\sin\delta\alpha + \sin\alpha\cos\delta\alpha}{\cos\alpha\cos\delta\alpha - \sin\alpha\sin\delta\alpha}\\ &= \frac{\sin\alpha + \cos\alpha\delta\alpha}{\cos\alpha - \delta\alpha\sin\alpha}\\ &= \frac{\sin\alpha}{\cos\alpha}\frac{1+\tan^{-1}\alpha\,\delta\alpha}{1-\tan\alpha\,\delta\alpha}\\ &\approx \tan\alpha\left(1+\tan^{-1}\alpha\ \delta\alpha\right)()\times(1+\tan\alpha\ \delta\alpha)()\\ &\approx \tan\alpha\left(1+\delta\alpha(\tan^{-1}\alpha+\tan\alpha)\right)\\ &\approx \tan\alpha\left(1+\frac{2\ \delta\alpha}{\sin 2\alpha}\right)\end{aligned} \tag{4}$$

Taking $\alpha = \theta/2$, we get that Eq. 3 simplifies to

$$\begin{aligned} \delta\eta &\approx -\ln\left(1 + \frac{\delta\theta}{\sin\theta}\right) \\ \delta\eta &\approx -\frac{1}{\sin\theta}\,\delta\theta \end{aligned} \tag{5}$$

(c) The differential solid angle is thus approximately:

$$\begin{aligned} \delta\Omega &= \sin\theta\,\delta\theta\,\delta\varphi \\ \delta\Omega &\approx -\sin^2\theta\delta\eta\,\delta\varphi \\ \delta\Omega &= -\frac{1}{\cosh^2\eta}\delta\eta\,\delta\varphi \end{aligned} \tag{6}$$

(d) As a consequence:

$$\begin{aligned} \frac{\delta N}{\delta\Omega} &\approx -\cosh^2\eta\,\frac{\delta N}{\delta\eta\delta\varphi} \\ \frac{\delta N}{\delta\Omega} &\approx \mathrm{cst}\times\cosh^2\eta \end{aligned} \tag{7}$$

Therefore the production rate is not constant vs η and increase exponentially at high η. The sketch would be a plot of $y(x) = \mathrm{cst}\times x^2$.

(e) By definition of a solid angle when gets that the differential surface is $\delta S/R^2 = \delta\Omega$ with R the radius of the sphere. In this case $\delta\Omega_{\mathrm{ch}} = 2.5\times 2.5/200^2 = 1.6\,10^{-4}$

(f) Since the detector is spherical we conclude that

$$\frac{\delta N}{\delta S} \approx \mathrm{cst}'\times\cosh^2\eta.$$

Thus the amount of particle collected by a single channel is exponentially growing with η. So the dose integrated over a single channel explodes with η. This has several consequences:

- Deterioration of detector is not isotropic but very large at high η while modest for lower η (central parts of the detectors).
- The occupancy also increases with η, *i.e.* that chances to get several particles from the pileup collisions in addition to an interesting particles are very high.

2. **Kinematics of $h \to aa \to 4\gamma$ decay**
We consider that the Higgs boson is produced at rest at the centre of the detector.

(a) Since the Higgs boson is emitted at rest the 2 ALPs have the same momentum $|\vec{p}_a|$ (with opposite direction), thus they have the same energy E_a. We thus get that

$$\begin{aligned} E_a &= \frac{m_h}{2} \\ |\vec{p}_a| &= \sqrt{E_a^2 - m_a^2} = \frac{m_h}{2}\times\sqrt{1-\frac{4\,m_a^2}{m_h^2}} \end{aligned} \tag{8}$$

(b) Noting $p_{\gamma i}$ the 4-momentum of the photon i, we get:

$$\begin{aligned} m_a^2 &= (p_{\gamma1}+p_{\gamma2})^2 = (p_{\gamma1})^2 + (p_{\gamma2})^2 + 2p_{\gamma1}.p_{\gamma2} \\ m_a^2 &= 2E_{\gamma1}\,E_{\gamma2}\,(1-\cos\Delta\theta_{\gamma\gamma}) \end{aligned} \tag{9}$$

(c) See the course, we have

$$\begin{aligned} E_a &= \gamma_a m_a \\ |\vec{p}_a| &= \gamma_a \beta_a m_a \end{aligned} \tag{10}$$

Thus

$$\begin{aligned} \gamma_a &= \frac{m_h}{2\,m_a} \\ \beta_a &= \frac{|\vec{p}_a|}{E_a} = \sqrt{1 - \frac{4\,m_a^2}{m_h^2}} \end{aligned} \tag{11}$$

(d) Assume $E_{\gamma 1} = E_{\gamma 2} \equiv E_\gamma$, the conservation of energy gives $2\,E_\gamma = E_a$ from which we get:

$$E_\gamma = \frac{m_h}{4}$$

(e) We assume $m_a \ll m_h$. Because $m_a^2 = 2E_\gamma^2\,(1 - \cos\Delta\theta_{\gamma\gamma})$, $(1 - \cos\Delta\theta_{\gamma\gamma}) = \frac{16\,m_a^2}{2\,m_h^2} \ll 1$. This implies that $\Delta\theta_{\gamma\gamma} \approx 0$. Developing to order 1 in $\Delta\theta_{\gamma\gamma}$, we get:

$$\begin{aligned} 1 - \cos\Delta\theta_{\gamma\gamma} &\approx (\Delta\theta_{\gamma\gamma})^2/2 \\ (\Delta\theta_{\gamma\gamma})^2 &\approx \frac{16\,m_a^2}{m_h^2} \\ \Delta\theta_{\gamma\gamma} &\approx \frac{4\,m_a}{m_h} \end{aligned} \tag{12}$$

(f) Assuming $m_a = 0.1$ GeV, what is the value of

$$\Delta\theta_{\gamma\gamma} \approx \frac{4\ 0.1}{125} \approx 0.003 \text{ rad},$$

while the angular size of a channel $\delta\varphi_{ch} = 2.5/200 \approx 0.0125$ rad. This means the two photons are not distinguishable and are likely to hit the same channel.

3. **Study of the decay** $a \to \gamma\gamma$

The required Feynman rules are given in the appendix, α represents the fine structure constant and $\frac{c_{\gamma\gamma}}{\Lambda}$ the dimensional coupling strength of the a particle to photons. For numerical applications, we will use $\frac{c_{\gamma\gamma}}{\Lambda} = 1$ TeV^{-1}.

We note the 4-momenta: p_a for a, k_1 and k_2 for the 2 photons.

(a) The matrix element is given by:

$$i\,\mathcal{M} = 4\,i\,\alpha\frac{C_{\gamma\gamma}}{\Lambda}\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\ \epsilon^*(k_1)\ \epsilon^*(k_2)$$

(b) The matrix element is Lorentz invariant and can be therefore be computed in any frame. We can also use the fact that: $m_a^2 = (k_1+k_2)^2 = k_1^2 + k_2^2 + 2\,k_1.k_2 = 2\,k_1.k_2$, because photons are massless so $k_1^2 = k_2^2 = 0$. Thus, we have that $k_1.k_2 = m_a^2/2$ and the matrix element is given by:

$$|\mathcal{M}_{a\gamma\gamma}|^2 = 4m_a^4\ |4\pi\alpha\frac{C_{\gamma\gamma}}{\Lambda}|^2$$

.

(c) See the course.

(d) Using the previous questions we can compute the partial width to $\gamma\gamma$ (in the centre of mass $k^* = m_a/2$):

$$\begin{aligned} \frac{d\Gamma}{d\Omega} &= m_a^3 |\alpha \frac{c_{\gamma\gamma}}{\Lambda}|^2 \\ \Gamma_{\gamma\gamma} &= 4\pi\alpha^2 \; |c_{\gamma\gamma}|^2 \; \frac{m_a^3}{\Lambda^2} \end{aligned} \tag{13}$$

(e) Since the lifetime of the particle in the lab. is increased by a factor γ_a (Lorentz time dilatation), the typical distance travelled by a is given by (τ_a is the a particle lifetime so $\tau_a = 1/\Gamma_a$)

$$\begin{aligned} L_a &= \gamma_a \; \beta_a \; \tau_a \\ L_a &= \gamma_a \; \beta_a \; \frac{1}{\Gamma_a} \end{aligned} \tag{14}$$

(f) The length of the detector is $L_{\text{det}} = 2$ m. What is the numerical value of L_{det} in GeV^{-1} unit? To transform length in energy, we use $\hbar c = 200\text{MeV.fm}$

$$L_{\text{det}} = \frac{2 \; 10^{15} \text{ fm}}{0.2 \text{ GeV.fm}} = 10^{16} \text{ GeV}^{-1}$$

(g) In order to detect the particle decay products, the particle needs to decay inside the volume of the detector so

$$L_a < L_{\text{det}}.$$

(h) Because the ALP solely decays to photons $\Gamma_a = \Gamma_{\gamma\gamma}$.

$$\begin{aligned} L_a &< L_{\text{det}} \\ \gamma_a \; \beta_a &< L_{\text{det}} \, \Gamma_{\gamma\gamma} \\ \frac{m_h}{2 \; m_a} &< L_{\text{det}} 8\pi\alpha^2 \; |c_{\gamma\gamma}|^2 \; \frac{m_a^3}{\Lambda^2} \\ m_a^4 &> m_h \frac{\Lambda^2}{|c_{\gamma\gamma}|^2} \frac{1}{L_{\text{det}}} \frac{1}{8\pi\alpha^2} \\ m_a^4 &> (0.125 \text{ TeV}) \; (1 \text{ TeV}^2)(10^{-19} \text{ TeV}) \frac{1}{8\pi(1/137)^2} \\ m_a^4 &> 93 \; 10^{-19} \text{ TeV}^4 \\ m_a^4 &> 0.093 \; 10^{-16} \text{ TeV}^4 \\ m_a &> (0.093)^{0.25} \; 10^{-4} \text{ TeV} \\ m_a &> 0.55 \; 0.1 \text{ GeV} \\ m_a &> 55 \text{ MeV} \end{aligned} \tag{15}$$

So in order for the particle to decay inside the volume of the detector its mass must be $m_a > 55$ MeV.