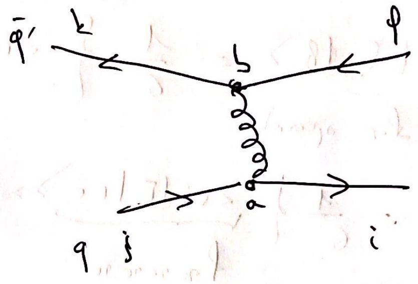
ES.6 correction

I. Color factors

1) See course.



i, j, k, l are color index

2) Matrix element.

$$\mathcal{M} = \left(\sum_a (c_i^\dagger t_a c_j)(c_k^\dagger t_a c_l) \right) \underbrace{\mathcal{M}_{QED}}_{\text{same as QED without coupling strenght}} \times g$$

$$\mathcal{M} = \underbrace{g_s^2}_{\text{coupling strengh}} \times \underbrace{f_{ijkl}}_{\text{color factor as defined in the course}} \times \underbrace{\mathcal{M}_{QED}}_{\text{same as QED } \mathcal{M}}$$

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$\Rightarrow$ The matrix element is deriving from a potential similar to QED

$$V_{q\bar{q}'} = \alpha_s \underbrace{f_{ijkl}}_{\text{color factor!}} V_{EM}$$

$F = f_{ijkl}$

3) for the octet element $|8\rangle$

$$|8\rangle = \frac{1}{\sqrt{6}} |r\bar{r} + g\bar{g} - 2b\bar{b}\rangle$$

The color factor part is:

$$\underbrace{\langle 8 | F_{c} | 8 \rangle}_{\text{" color operator}} = \frac{1}{6} \{ \langle r\bar{r} + g\bar{g} - 2b\bar{b} | F | r\bar{r} + g\bar{g} - 2b\bar{b} \rangle \}$$

$$= \frac{1}{6} \{ \underbrace{\langle r\bar{r} | F | r\bar{r} \rangle}_{f_{xxxx}} \times (\underset{\langle r\bar{r}|r\bar{r}}{1} + \underset{g\bar{g}|g\bar{g}}{1} + \underset{b\bar{b}|b\bar{b}}{4})$$

$$+ \underset{f_{gxxg}}{\langle r\bar{r} | F | g\bar{g} \rangle} \times (\underset{r\bar{r}/g\bar{g}}{1} - \underset{g\bar{r}/b\bar{b}}{2} + \underset{g\bar{g}/r\bar{r}}{1} - \underset{g\bar{g}/b\bar{b}}{2} - \underset{b\bar{b}/r\bar{r}}{2} - \underset{b\bar{b}/g\bar{g}}{2}) \}$$

$$= \frac{1}{6} \{ f_{xxxx} \, 6 + f_{gxxg} \, -6 \}$$

$$= 1 \times \{ \underbrace{f_{xxxx}}_{+1/3} - \underbrace{f_{gxxg}}_{1/6} \} = -\frac{1}{6} < 0$$

thus a repulsive force

4) $|8\rangle \to |0\rangle$ ie changing of color multiplet should be impossible

$$\langle 0 | F | 8 \rangle = \frac{1}{3\sqrt{2}} \{ \langle r\bar{r} + g\bar{g} + b\bar{b} | F | r\bar{r} + g\bar{g} - 2b\bar{b} \rangle \}$$

$$= \frac{1}{3\sqrt{2}} \{ \langle r\bar{r} | F | r\bar{r} \rangle \times (1 + 1 - 2)$$

$$\langle r\bar{r} | F | g\bar{g} \rangle \times (\underset{r\bar{r}/g\bar{g}}{1} - \underset{r\bar{r}/b\bar{b}}{2} + \underset{g\bar{g}/r\bar{r}}{1} - \underset{g\bar{g}/b\bar{b}}{2} + \underset{b\bar{b}/r\bar{r}}{1} + \underset{b\bar{b}/g\bar{g}}{1}) \}$$

$$\langle 0 F | 8 \rangle = 0$$

Changing of multiplet is indeed impossible (2)

$\langle o | F | 8 \rangle = 0$

5) $|o\rangle \to |o\rangle$ was done in course

$\langle o | F | o \rangle$ is > 0, attractive force

II. Nucleon momentum.

1) The total p momentum is the sum of u, d, g momenta.

$$P_p = \int x P_p f_u^p(x)\, dx + \int x P_p f_d^p(x)\, dx + \int x P_p f_g^p\, dx$$

with P_p the proton momentum

$$P_p = P_p \cdot (\varepsilon_u^p + \varepsilon_d^p + \varepsilon_g^p)$$

$$\Rightarrow \quad \varepsilon_u^p + \varepsilon_d^p + \varepsilon_g^p = 1$$

2) Isospin $\Rightarrow$ $f_u^n = f_d^p$

$$f_d^n = f_u^p \qquad f_g^n = f_g^p$$

We also get $\varepsilon_u^n + \varepsilon_d^n + \varepsilon_g^n = 1$

$$\Rightarrow \quad \varepsilon_u + \varepsilon_d + \varepsilon_g = 1$$

3) $F_2^p(x) = \sum_q Q_q^2 \, f_q^p(x)$

$$\int x F_2^p(x)\,dx = \left(\frac{2}{3}\right)^2 \int f_u^p(x)\, x\, dx + \left(\frac{1}{3}\right)^2 \int x\, f_d^p(x)\, dx$$

$$= \left(\frac{2}{3}\right)^2 \varepsilon_u^p + \left(\frac{1}{9}\right) \varepsilon_d^p = \frac{4}{9}\varepsilon_u + \frac{1}{9}\varepsilon_d$$

4) $$\int x F_2^n(x)\,dx = \left(\frac{2}{3}\right)^2 \varepsilon_u^n + \frac{1}{9}\varepsilon_d^n \underset{\uparrow \atop \text{isospin}}{=} \frac{4}{9}\varepsilon_d + \frac{1}{9}\varepsilon_u$$

5)
$$4\varepsilon_u + \varepsilon_d = 9 \times 0.18$$
$$4\varepsilon_d + \varepsilon_u = 9 \times 0.12$$

$15\varepsilon_d = 4\times 9 \times 0.12 - 9 \cdot 0.18$
$\Rightarrow \varepsilon_d = 0.18$

$$\Rightarrow \begin{cases} \varepsilon_u = 0.36 \\ \varepsilon_d = 0.18 \end{cases}$$

$$\Rightarrow \boxed{\varepsilon_g = 1 - \varepsilon_u - \varepsilon_d = 0.46}$$

About half the momentum of the proton is carried out by gluons. This was a proof of a missing piece in partons: the gluons