Cross section

Master formula of cross section

In PDG 2020: page 253, formula 48.27

It contains

$$d\sigma = \underset{\substack{\uparrow \\ 4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2} \\ \text{Initial st. kin.}}}{\frac{1}{F}} |\mathcal{M}|^2 \overbrace{(2\pi)^4 d\Phi_n}^{\text{Final st. kin. : phase space (n body)}}$$

$$F = 4 p_i^* \sqrt{s}$$

$$1 \xrightarrow{p_i^*} \xleftarrow{p_i^*} 2 \quad \sqrt{s} \qquad \text{(CM)}$$

(For $d\Gamma$ $F \to 2m_1$)

$1+2 \to 3+4$ (2 body final state)

Integration of $(2\pi)^4 d\Phi_n$ to get 2 dof by taking into account the constraints ($\Rightarrow$ getting rid of the δ function)

$$(2\pi)^4 d\Phi_n \to \frac{1}{(4\pi)^2} \frac{p_f^*}{\sqrt{s}} d\Omega$$

So that

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{s} \frac{p_f^*}{p_i^*} |\mathcal{M}|^2$$

$\mathcal{M} = \langle f|T|i\rangle$ (matrix element)

In $|i\rangle$ and $|f\rangle$ (initial and final states) the spin states are well defined ($\mathcal{M}$ may depend on them)

If $d\sigma$ is computed for several states of spins ($|i\rangle$ and $|f\rangle$) we must

→ <u>Sum</u> the $d\sigma$ from different <u>final states</u>

→ <u>Average</u> $d\sigma$ from all possible spins in the <u>initial state</u> (a particle in the initial state is a statistical mixture of spin states

$-s_1, -s_1+1, \dots s_1$ etc.

This results in "weigts" like $\frac{1}{(2s_1+1)}$ $\frac{1}{(2s_2+1)}$

→ If ther are k identical particles in the final state this must be devided by $k!$ (holds for each group of identical particles)

Rem: ① In case of polarised beams there is only one state of spin in the initial state.

② If $\langle i|T|f\rangle$ does not depend on the spin of the initial state it is useless to do the average.