1 Part A (10 points total)

1.1 Question A1 (2.5 points)

A beam of muons with momentum $400\,\text{MeV}/c$ passes through a water tank of depth 1 m (perpendicular to the beam).

(a) Estimate the mean energy loss in MeV of each muon (within a factor of two), explaining your reasoning and noting any assumptions made. [**1.5 points**]

(b) Explain briefly and qualitatively why $400\,\text{MeV}/c$ muons can pass through the water tank, whereas both alpha particles and electrons of similar momentum are likely to be absorbed. (Detailed calculations are not required.) [**1 point**]

(a) $p = 400\,\text{MeV}/c$ so these are MIPs—for a muon, the energy loss minimum is between $300\,\text{MeV}/c$ and $400\,\text{MeV}/c$, and it's a shallow minimum, so these will qualify. For any singly charged MIP, stopping power in any material is

$$\frac{1}{\rho}\left\langle\frac{dE}{dx}\right\rangle \approx (1-2)\,\text{MeV}\,\text{g}^{-1}\,\text{cm}^2.$$

In this case the material is water, so $\rho \simeq 1\,\text{g}\,\text{cm}^{-3}$. This means that for our muons,

$$\left\langle\frac{dE}{dx}\right\rangle \approx \frac{1}{\rho}\times(1-2)\,\text{MeV}\,\text{g}^{-1}\,\text{cm}^2 \approx (1-2)\,\text{MeV}\,\text{cm}^{-1}.$$

The tank is 1 m (100 cm) deep, so the energy loss is about (100–200) MeV. This is significant, between a quarter and a half of the initial momentum, but not so big that the muons are no longer MIP-like.

(b) Muons of this momentum, 400 MeV are minimum-ionising particles, but this is not true either of electrons or of alphas.
(i) In the case of electrons, $\beta\gamma$ is far *above* the MIP plateau, and there is additional loss through radiation effects, i.e. Bremsstrahlung. (Electrons also lose more energy from Coulomb scattering on atomic electrons, since unlike muons they are not much heavier than those atomic electrons.) The net effect is that energy loss is greater and the particles are absorbed more quickly. (This difference is not enormous at this particular energy scale: the expected range of 400 MeV electrons in water is a bit less than 1m.)
(ii) In the case of alpha particles, there are two key effects. One is that they are heavier than muons by a factor of about 40, so $\beta\gamma$ is much lower, and thus $p = 400\,\text{MeV}$ alpha particles are well below the MIP region and have much greater energy loss. The other is that an alpha particle has an electric charge of $+2$, and Coulomb energy losses go as z^2 (per Bethe-Bloch), so this gives another factor of 4 increase.

1.2 Question A2 (2 points)

You are investigating whether a crystal scintillator can be used as the electromagnetic calorimeter for various different proposed e^+e^- detectors. The two use cases being considered are

(i) photons of typical energy 80 GeV from Higgs decays at a $\sqrt{s} \approx 250\,\text{GeV}$ collider studying electroweak physics;

(ii) photons of typical energy 800 MeV from π^0 decays at a $\sqrt{s} \approx 10\,\text{GeV}$ collider studying flavour physics.

Given that for this scintillator material the critical energy is $E_c = 17.4\,\text{MeV}$ and the radiation length is $X_0 = 2.59\,\text{cm}$, how deep would the crystal calorimeter need to be to fully absorb the shower in each case? **[2 points]**

For NaI, $E_c = 17.4\,\text{MeV}$ and $X_0 = 2.59\,\text{cm}$. Shower propagates with number of particles N after t radiation lengths given by $N(t) = 2^t$, and thus each individual particle has mean energy $E(t) = E_0/N(t) = E_0 2^{-t}$. Shower propagation will stop when $E(t) = E_c$. Thus, at end of shower:

$$\begin{aligned}
E(t) &= E_0 2^{-t} = E_c \\
\Rightarrow \ln(2^{-t}) &= \ln(E_c/E_0) \\
\Rightarrow t &= \ln(E_0/E_c)/\ln(2) \\
\Rightarrow D &= tX_0 = X_0 \ln(E_0/E_c)/\ln(2)
\end{aligned}$$

Evaluating this for $E_0 = 80\,\text{GeV}$, we get:

$$D = (2.59\,\text{cm}) \times \ln(80\,\text{GeV}/17.4\,\text{MeV})/\ln(2) = 31.5\,cm$$

$$t = \ln(80\,\text{GeV}/17.4\,\text{MeV})/\ln(2) = 12.17$$

and for $E_0 = 800\,\text{MeV}$, we get:

$$D = (2.59\,\text{cm}) \times \ln(800\,\text{MeV}/17.4\,\text{MeV})/\ln(2) = 14.3\,cm$$

$$t = \ln(800\,\text{MeV}/17.4\,\text{MeV})/\ln(2) = 5.5$$

So on paper we need about 32 cm for the high-energy case (i), and about 15 cm for the low-energy case (ii). There will be some stragglers so we can give ourselves a little margin, so say: 34–37 cm and 17-19 cm. [Note that questions of whether, for example, the shower starts at the beginning of the calorimeter or after one radiation length make very little difference in the end: a few cm. Note also that the length required scales logarithmically with the energy, so that at least in principle a relatively compact calorimeter can work even for high energies.]

1.3 Question A3 (1 point)

A nuclear reaction generates neutrons of energy 1 MeV. Shielding is placed around the experiment to thermalise the neutrons. Suggest a suitable material for the shielding, giving a brief (physics) justification for your answer. **[1 point]**

Note that the question asks for a **physics** argument. The most obvious one is that low-A materials absorb neutron energy more efficiently, thus good materials contain hydrogen atoms (or other low-A elements), e.g. water or hydrocarbons like plastic or paraffin. One could also argue for materials containing lithium (Z=3) or boron (Z=5) or carbon (Z=6), which still have fairly low A. On the other hand, intermediate-A and high-A materials (e.g. lead, tungsten, iron) don't work well at thermalising neutrons and would be a poor choice.

1.4 Question A4 (4.5 points)

You are selecting a photodetector for a new experiment that will produce scintillation light. A number of different PMT-based sensor designs are available, with different quantum efficiencies.

(a) Define the term "quantum efficiency". [**1 point**]

(b) Explain why the quantum efficiency is significantly less than 100%, and why it varies with wavelength and between different sensors. [**2 points**]

One particular photodetector would be a good match for the requirements of the experiment, except that its quantum efficiency is optimised for longer wavelengths and does not cover the range expected for the scintillation light. A colleague proposes the use of wavelength-shifting fibres.

(c) Outline the (physics) mechanism by which this solution would work. Explain whether it would still work if the situation were reversed, i.e. if the quantum efficiency were optimised for wavelengths shorter than the scintillation light. [**1.5 points**]

(a) QE: Probability for photon to interact with photocathode AND emit an electron that successfully gets out of the photocathode into the vacuum tube.

(b) (i) Why QE is significantly less than 100%: For a finite thickness of material, there is a nonzero probability for the photon to escape without undergoing a photoelectric effect, and a nonzero probability for the photoelectron to be recaptured in the material. Moreover, the two effects are in tension: more photocathode material will increase the probability of the initial photoelectric effect but will also increase the electron absorption probability, and vice versa. (Note also that both photoelectric cross-section and dE/dx increase with Z, although not in the same way.)
(ii) Why QE varies with wavelength and between sensors: The photoelectric effect depends heavily on photon energy: a minimum energy is required to liberate electrons from a particular energy level, but the cross-section then drops rapidly with energy above that threshold. This pattern might repeat a few times depending on the atomic structure (especially for a compound/mixture/alloy), but the end result is that there will be a minimum and a maximum useful energy (i.e. a max and a min wavelength). Different sensor materials will lead to different useful ranges (and different overall probabilities, since the two effects mentioned in (i) depend on the material).

(c) See course slides or a textbook on how wavelength-shifting materials work (mixture of different scintillator materials, so that the first emits scintillation light at E_1, the second can absorb at E_1 and then re-emit at $E_2 < E_1$, a third material can absorb at E_2 and then re-emit at $E_3 < E_2$, potentially with additional cycles). Note that the optical photon energy steps down with each cycle (with the rest of the energy being lost as phonons, kinetic energy, other transitions between energy levels that emit soft photons, etc), so it can be used to shift to *longer* wavelengths, but cannot be used to shift to shorter (higher-energy) wavelengths.