From nuclei to stars
Theoretical course

NPAC 2019-2020

Final exam 05/02/2020

1. Nuclear interaction
   a. What is Yukawa's model of nuclear interaction and what is the logic behind it?
   b. In which way modern nucleon-nucleon potentials improve on Yukawa's idea?
2. Prove that the state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\mu\nu\rangle + |\mu\alpha\rangle)$ is an eigenstate of the particle-number operator $N = \sum_i a_i^+ a_i$, but not an eigenstate of all individual components $a_i^+ a_i$.
3. Let us consider a set of creation and annihilation operators $\{a_\alpha^+; a_\alpha\}$ and their corresponding vacuum $|\Phi_0\rangle$.
   a. What property does $|\Phi_0\rangle$ have under the action of a_α?

   Making use of Wick's theorem with respect to $|\Phi_0\rangle$, compute the vacuum expectation value of

   b. The operator $a_\alpha a_\beta a_\gamma^+ a_\delta^+$;
   c. The operator $a_\alpha^+ a_\beta^+ a_\gamma a_\delta a_\lambda a_\mu^+$.
4. Hartree-Fock
   a. Can nucleons be considered as independent particles and why?
   b. Explain schematically in what consists the Hartree-Fock method.
   c. In which sense Hartree-Fock is a *self-consistent* approximation?
5. Symmetry breaking
   a. When approximating the solution of the many-body Schrödinger equation, why one would be interested in using a wave function that does not have the same symmetries of the exact one?
   b. Which transformation of the creation and annihilation operators introduces particle-number breaking? What are the consequences on the density matrices?
6. Derive the normal-ordered form of the Hamiltonian

$$H = \sum_{\alpha\beta} t_{\alpha\beta}\, a_\alpha^+ a_\beta + \left(\frac{1}{2!}\right)^2 \sum_{\alpha\beta\gamma\delta} \bar{v}_{\alpha\beta\gamma\delta}\, a_\alpha^+ a_\beta^+ a_\delta a_\gamma$$

   with respect to a generic *Bogolyubov* vacuum $|\Phi\rangle$ (which is *not* the vacuum of the $\{a_\alpha^+; a_\alpha\}$). Recall that

   - The matrix elements of the two-body operator appear in their antisymmetrised version:

$$\bar{v}_{\alpha\beta\gamma\delta} \equiv v_{\alpha\beta\gamma\delta} - v_{\alpha\beta\delta\gamma} \, .$$

   - One can make use of the density matrices

$$\rho_{\alpha\beta} \equiv \langle\Phi|a_\beta^+ a_\alpha|\Phi\rangle \, , \qquad \kappa_{\alpha\beta} \equiv \langle\Phi|a_\beta a_\alpha|\Phi\rangle \, .$$