From nuclei to stars
Theoretical course

NPAC 2020-2021

Final exam 21/01/2021

1. Why we describe atomic nuclei in terms of nucleonic degrees of freedom and not directly in terms of quarks and gluons?

2. Partial-wave channels of the nucleon-nucleon interaction

   a) Prove that total antisymmetry of the two-nucleon wave function implies $(-1)^{L+S+T} = -1$, where L, S and T denote respectively the orbital angular momentum, spin and isospin of the nucleon pair.

   b) Which between L, S, T and J (the total angular momentum) are conserved by nucleon-nucleon interactions?

   c) Point b) implies that the nuclear interaction can be decomposed into spin-isospin-angular momentum channels ("partial waves"). Point a) implies that not all partial waves are allowed. Using the spectroscopic notation, list three allowed and three disallowed partial waves.

3. What is the scattering length? What do nucleon-nucleon scattering lengths tell us about the nucleon-nucleon interaction?

4. Show that $W|\mu\nu\rangle = 0$, where W is a generic three-body operator and $|\mu\nu\rangle$ a generic two-body state.

5. Given the string of creation and annihilation operators $a_b a_c^+ a_d^+ a_e$

   a) Apply Wick's theorem with respect to a vacuum state $|\Phi_1\rangle$ that is the vacuum of the operators $\{a^+, a\}$ themselves.

   b) Compute the expectation value $\langle\Phi_1|a_b a_c^+ a_d^+ a_e|\Phi_1\rangle$.

   c) Apply Wick's theorem with respect to a generic Slater determinant $|\Phi_2\rangle$ (which is *not* the vacuum of the operators $\{a^+, a\}$ themselves).

   d) Compute the expectation value $\langle\Phi_2|a_b a_c^+ a_d^+ a_e|\Phi_2\rangle$.