From nuclei to stars
Theoretical course

NPAC 2022-2023

Mid-term exam 16/11/2022

1. Categorisation of atomic nuclei.

   a. Explain what is the difference between stable, unstable and unbound nuclei.

   b. How many systems (i.e., different combinations of protons and neutrons) exist in nature for each of these three categories?

   c. Name an example for each of the three categories.

2. Imagine that in a few years a supercomputer with a total memory of 1 ZB ($= 10^{21}$ bytes) is available. What is the heaviest nucleus with $N = Z$ that can be computed via a full configuration-interaction calculation on this machine?

   To make your estimate:

   - Use a one-body basis made of $3Z$ states.
   - Suppose that the Hamiltonian matrix contains $r^{1.2}$ non-zero elements, with r being the number of rows.
   - Work in *double precision*, i.e., assume that 8 bytes are required to store each matrix element.

3. Bases of $\mathcal{H}_A^F$.

   a. What must be changed in the construction of the many-body Hilbert space when one considers *indistinguishable* particles?

   b. Explain how to construct a product-state basis for the fermionic Hilbert space $\mathcal{H}_A^F$.

   c. Write a basis of $\mathcal{H}_A^F$ for $A = 3$ where states are eigenstates of the one-body linear momentum.

4. The so-called *ab initio* many-body problem can be divided into two sub-problems, i.e., (i) the modelling of the nuclear Hamiltonian and (ii) the solution of the many-body Schrödinger equation.

   a. What characterises an *ab initio* nuclear Hamiltonian?

   b. What characterises an *ab initio* solution to the many-body Schrödinger equation?